MPM2D – Unit 7 – Analytical Geometry
Review Worksheet #1

Section 1 – Line Segments

1. C(1,10) D(5,2)

a) $M_{CD} = M\left(\frac{x_C+x_D}{2}, \frac{y_C+y_D}{2}\right)$

$= M\left(\frac{1+5}{2}, \frac{10+2}{2}\right)$

$= M(3, 6)$

b) $m = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{2-10}{5-1}$

$= \frac{-8}{4}$

$= -2$

Right Bisector: $m_\perp = +1/2$

$y = mx + b$

$6 = +1/2(3) + b$

$6 = +3/2 + b$

$12/2 - 3/2 = b$

$b = 9/2$

$y = 1/2x + 9/2$

(7,8) – check

$y = -\frac{1}{2}x + 9/2$

$8 = \frac{1}{2}(7) + 9/2$

$8 = 7/2 + 9/2$

$8 = 16/2$

$8 = 8$

2. C(-3,5) and M(6,-1) D = ?

$M_x = \frac{x_C+x_D}{2}$ $M_y = \frac{y_C+y_D}{2}$

$6 = \frac{-3+x_D}{2}$ $-1 = \frac{5+y_D}{2}$

$12 = -3 + x_D$ $-2 = 5 + y_D$

$x_D = 15$ $y_D = -7$

∴ D) (15, -7)

3. H(9,-7) and M(-1,-3) I?

$M_x = \frac{x_H+x_I}{2}$ $M_y = \frac{y_H+y_I}{2}$

$-1 = \frac{9+x_I}{2}$ $-3 = \frac{-7+y_I}{2}$

$-2 = 9 + x_I$ $-6 = -7 + y_I$

$x_I = -11$ $y_I = 1$

I (-11, 1)

Section 2 - Circles

4. Centre (0,0) Endpt D(5,12)

a) radius

$L_{OD}^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$= (5-0)^2 + (12-0)^2$

$= 5^2 + 12^2$

$= 25 + 144$

$= 169$

$L_{OD} = \sqrt{169}$

$= 13$ units

b) $x^2+y^2 = r^2$

$x^2+y^2 = (13)^2$

$x^2+y^2 = 169$

c) $A = \pi r^2$

$= \pi(13)^2$

$= 169\pi$ units^2

5. $x^2+y^2 = 36$ $d = 12$ units

$r^2 = 36$ This shows the radius of the circle is 6 units

$\sqrt{r^2} = \sqrt{36}$ The diameter is twice the radius and

$r = \pm 6$ therefore is 12 units

6a) $x^2+y^2 = 64$

$(-7)^2+(4)^2 = 64$

$49+16 = 64$

$65 = 64$

$\therefore x^2+y^2 > r^2$

$\therefore$ Pt $(-7,4)$ is outside the circle

b) $x^2+y^2 = 100$

$(8)^2+(-6)^2 = 100$

$64+36 = 100$

$100 = 100$

$\therefore x^2+y^2 = r^2$

$\therefore$ Pt $(8,-6)$ is on the circumference of the circle

c) $x^2+y^2 = 144$

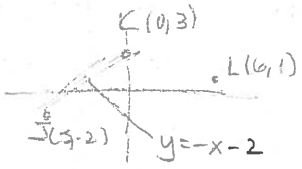
$(-2)^2+(10)^2 = 144$

$4+100 = 144$

$104 = 144$

$\therefore x^2+y^2 < r^2$

$\therefore$ The Pt $(-2,10)$ is inside the circle



7. J(-5,-2) K(0,3) L(6,1) Right Bisector of JK : $y=-x-2$

$m_{KL}=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{1-3}{6-0}$

$=\frac{-2}{6}$

$=-1/3$

$m_\perp = 3$

$M_{KL}=M\left(\frac{x_K+x_L}{2},\frac{y_K+y_L}{2}\right)$

$=M\left(\frac{0+6}{2},\frac{3+1}{2}\right)$

$=M(3,2)$

$y=mx+b$

$2=3(3)+b$

$2=9+b$

$b=-7$

$y=3x-7$

Centre = POI

$y=-x-2$ ①

$y=3x-7$ ②

① = ②

$-x-2=3x-7$

$-x-3x=-7+2$

$-4x=-5$

$x=5/4$

Sub $x=5/4$ into ①

$y=-x-2$

$=-5/4-2/1$

$=-5/4-8/4$

$=-13/4$

∴ The centre of the circle is $\left(\frac{5}{4},\frac{-13}{4}\right)$

8. G(1,2) H(4,-7) I(5,0)

$m_{GH}=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{-7-2}{4-1}$

$=\frac{-9}{3}$

$=-3$

$m_\perp = 1/3$

$M(x,y)=M\left(\frac{x_G+x_H}{2},\frac{y_G+y_H}{2}\right)$

$=M\left(\frac{1+4}{2},\frac{2+(-7)}{2}\right)$

$=M(5/2,-5/2)$

$y=mx+b$

$-5/2=1/3(5/2)+b$

$-5/2=5/6+b$

$-15/6-5/6=b$

$b=-20/6 \rightarrow b=-10/3$

$m_{HI}=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{0-(-7)}{5-4}$

$=7$

$m_\perp=-1/7$

$M(x,y)=M\left(\frac{x_H+x_I}{2},\frac{y_H+y_I}{2}\right)$

$=M\left(\frac{4+5}{2},\frac{-7+0}{2}\right)$

$=M(9/2,-7/2)$

$y=mx+b$

$-7/2=-1/7(1/2)+b$

$-7/2=-9/14+b$

$-49/14+9/14=b$

$b=-40/14$

$b=-20/7$

$y=\frac{1}{3}x-10/3$ ①

$y=-\frac{1}{7}x-20/7$ ②

① × 3: $3y=x-10$ ③

② × 7: $7y=-x-20$ ④

③ + ④ $10y=-30$

$y=-3$

Sub $y=-3$ into ③

$3(-3)=x-10$

$-9+10=x$

$x=1$

∴ The centre of the circle is $(1,-3)$

9. A(-2,-1) B(4,3) C(12,-1)

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3-(-1)}{4-(-2)}$

$= \frac{4}{6}$

$= \frac{2}{3}$

$m_\perp = -3/2$

$M(x,y) = M\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$

$= M\left(\frac{-2+4}{2}, \frac{-1+3}{2}\right)$

$= M(1,1)$

$y = mx + b$

$1 = -3/2(1) + b$

$1 = -3/2 + b$

$3/2 + 3/2 = b$

$b = 5/2$

$y = -\frac{3}{2}x + \frac{5}{2}$

$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1-3}{12-4}$

$= \frac{-4}{8}$

$= -\frac{1}{2}$

$m_\perp = 2$

$M(x,y) = M\left(\frac{x_B + x_C}{2}, \frac{y_B + y_C}{2}\right)$

$= M\left(\frac{4+12}{2}, \frac{3+(-1)}{2}\right)$

$= M(8,1)$

$y = mx + b$

$1 = 2(8) + b$

$1 = 16 + b$

$-15 = b$

$y = 2x - 15$

$y = -3/2x + 5/2$ ①

$y = 2x - 15$ ②

① × 2: $2y = -3x + 5$ ③

② × -2: $-2y = -4x + 30$ ④

③ + ④ $0 = -7x + 35$

$7x = 35$

$x = 5$

Sub $x = 5$ into ②

$y = 2(5) - 15$

$= 10 - 15$

$= -5$

∴ The centre of the circle is (5, -5).

10. A(-6,-1) B(2,3) C(20,-3)

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{3-(-1)}{2-(-6)}$
$= \frac{4}{8}$
$= \frac{1}{2}$

$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-3-3}{20-2}$
$= \frac{-6}{18}$
$= \frac{-1}{3}$

$m_{\perp} = -2$

$m_{\perp} = 3$

$M(x,y) = M\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$
$= M\left(\frac{-6+2}{2}, \frac{-1+3}{2}\right)$
$= M(-2, 1)$

$M(x,y) = M\left(\frac{x_B + x_C}{2}, \frac{y_B + y_C}{2}\right)$
$= M\left(\frac{2+20}{2}, \frac{3+(-3)}{2}\right)$
$= M(11, 0)$

$y = mx + b$
$1 = -2(-2) + b$
$1 = 4 + b$
$b = -3$

$y = -2x - 3$

$y = mx + b$
$0 = 3(11) + b$
$0 = 33 + b$
$b = -33$

$y = 3x - 33$

$y = -2x - 3$ ①
$y = 3x - 33$ ②

① = ② $-2x - 3 = 3x - 33$
$-2x - 3x = -3 - 33$
$-5x = -30$
$x = 6$

Sub $x = 6$ into ①
$y = -2(6) - 3$
$= -12 - 3$
$= -15$

∴ The centre of the circle is $(6, -15)$.

11. $M(x,y) = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
$= M\left(\frac{8+8}{2}, \frac{-6+6}{2}\right)$
$= M(8, 0)$

A (8,6)
B (8,0)

$r^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$= (8-8)^2 + (6-0)^2$
$= 0^2 + 6^2$
$= 36$
$r = \sqrt{36}$
$r = 6$ units

$A = \pi r^2$
$= \pi(6)^2$
$= 36\pi$ units^2

Section 3 - Triangles and Quadrilaterals

12. M(-4,-1) N(7,6) O(10,-3) $L_{MN} = \sqrt{170}$ $L_{NO} = \sqrt{90}$ $L_{OM} = \sqrt{200}$

$m_{MN} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-(-1)}{7-(-4)} = \frac{7}{11}$

$m_{NO} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-6}{10-7} = \frac{-9}{3} = -3$

$m_{MO} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-(-1)}{10-(-4)} = \frac{-2}{14} = -\frac{1}{7}$

not isosceles or equilateral

⇓

find perp. bisector

$m_{NO} = -3$

$y = mx + b$

$y = -3x + b$

$6 = -3(9) + b$

$6 = -21 + b$

$6 + 21 = b$

$b = 27$

$y = -3x + 27$

$m \perp = 1/3$, M(-4,-1)

$y = mx + b$

$y = \frac{1}{3}x + b$

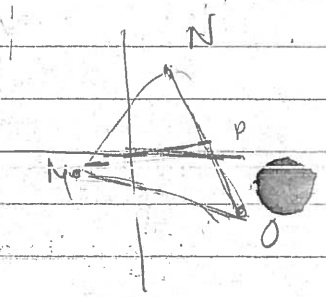
$-1 = \frac{1}{3}(-4) + b$

$-1 = -\frac{4}{3} + b$

$-\frac{3}{3} + \frac{4}{3} = b$

$b = \frac{1}{3}$

$y = \frac{1}{3}x + \frac{1}{3}$



$y = -3x + 27$ (1)

$y = \frac{1}{3}x + \frac{1}{3}$ (2)

(2)×3 $3y = x + 1$ (3)

(3)×3 $9y = 3x + 3$ (4)

$y = -3x + 27$ (1)

(4)+(1) $10y = 30$

$y = 3$

Sub $y = 3$ into (1)

$3 = -3x + 27$

$3 - 27 = -3x$

$-24 = -3x$

$x = 8$

POI IS (8,3) P, M(-4,-1)

$L_{MP}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$= [8-(-4)]^2 + [3-(-1)]^2$

$= (12)^2 + (4)^2$

$= 144 + 16$

$L_{MP} = \sqrt{160}$ units

$A = bh/2$

$= \frac{L_{NO} \cdot L_{MP}}{2}$

$= \frac{\sqrt{90} \cdot \sqrt{160}}{2}$

$= 60$ units^2

13. P(-9,-7) Q(4,6) R(10,-3)

$M_{QR} = M\left(\frac{x_Q+x_R}{2}, \frac{y_Q+y_R}{2}\right)$

$= M\left(\frac{4+10}{2}, \frac{6+(-3)}{2}\right)$

$= M(7, 3/2)$

$m_{QR} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-3-6}{10-4}$

$= \frac{-9}{6}$

$= -3/2$

$m_\perp = 2/3$

$y = mx+b$

$3/2 = 2/3(7/1)+b$

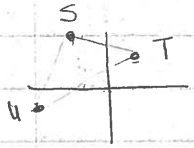
$3/2 = 14/3 + b$

$9/6 - 28/6 = b$

$b = -19/6$

$y = 2/3x - 19/6$

14. S(-4,8) T(2,6) U(-7,-1)



$m_{SU} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{8-(-1)}{-4-(-7)}$

$= \frac{19}{3}$

$= 3$

$m_{ST} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{6-8}{2-(-4)}$

$= \frac{-2}{6}$

$= -1/3$

$SU \perp ST$

base: $L_{ST}^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$= (2-(-4))^2 + (6-8)^2$

$= (6)^2 + (-2)^2$

$= 36+4$

$L_{ST} = \sqrt{40}$ units

height: $L_{SU}^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$= [-4-(-7)]^2 + [8-(-1)]^2$

$= (3)^2 + (9)^2$

$= 9+81$

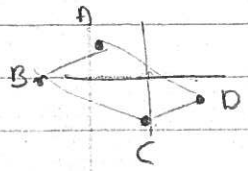
$L_{SU} = \sqrt{90}$ units

$A = \frac{bh}{2}$

$= \frac{\sqrt{40} \cdot \sqrt{90}}{2}$

$= 30$ units^2

15. A(-1,2) B(-4,0) C(0,-6) D(3,-4)



$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{-1-(-4)} = \frac{2}{3}$

$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-0}{0-(-4)} = \frac{-6}{4} = -\frac{3}{2}$

$AB \perp BC$

$L_{AB}^2 = (x_2-x_1)^2 + (y_2-y_1)^2$
$= (-1-(-4))^2 + (2-0)^2$
$= 3^2 + 2^2$
$= 9+4$
$L_{AB} = \sqrt{13}$

$L_{BC}^2 = (x_2-x_1)^2 + (y_2-y_1)^2$
$= [0-(-4)]^2 + [-6-0]^2$
$= 4^2 + (-6)^2$
$= 16+36$
$L_{BC} = \sqrt{52}$

$A = bh$
$= L_{BC} \cdot L_{AB}$
$= (\sqrt{52})(\sqrt{13})$
$= 26 \text{ units}^2$

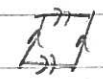
16. A triangle is equilateral if all 3 side lengths are equal.

b A triangle is isosceles if two of the three side lengths are equal

c. A triangle is scalene if no side lengths are equal.

17. A square has 4 equal side lengths and two lines are perpendicular
A rectangle has 2 sets of equal lengths and two lines are perpendicular

b. A rectangle has 2 sets of equal sides lengths that are perpendicular to each other.
A parallelogram has 2 sets of parallel lines



18. P(9,4) Q(3,-5) R(-3,-1)

$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-5)}{9-3} = \frac{9}{6} = 3/2$

$m_{QR} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-(-1)}{3-(-3)} = \frac{-4}{6} = -2/3$

$m_{PR} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-1)}{9-(-3)} = \frac{5}{12}$

∵ PQ ⊥ QR ∴ It is a right triangle

19. A(7,-5) B(-1,6) C(12,2)

$L_{AB}^2 = (x_2-x_1)^2 + (y_2-y_1)^2 = [7-(-1)]^2 + [-5-6]^2 = (8)^2 + (-11)^2 = 64 + 121 = 185$

$L_{AB} = \sqrt{185}$ units

$L_{BC}^2 = (x_2-x_1)^2 + (y_2-y_1)^2 = [12-(-1)]^2 + [2-6]^2 = (13)^2 + (-4)^2 = 169 + 16 = 185$

$L_{BC} = \sqrt{185}$

∵ $L_{AB} = L_{BC}$
∴ It is an isosceles triangle

20. D(-2,3) E(6,7) F(24,-20) G(14,-25)

$m_{DE} = 4/8 = 1/2$ $m_{EF} = -27/18 = -3/2$

$m_{DG} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-25-3}{14-(-2)} = \frac{-28}{16} = \frac{-7}{4}$

$m_{FG} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-20-(-25)}{24-14} = \frac{5}{10} = \frac{1}{2}$

Since two of the slopes are equal, there are 2 parallel lines. The other 2 lines are not parallel, making this a trapezoid.

21. D(-8,2) E(20,16) F(32,1) G(6,-12)

$m_{DE} = 14/28 = 1/2$ $\quad m_{EF} = -15/12 = -5/4$

$m_{FG} = \frac{y_2-y_1}{x_2-x_1} = \frac{1-(-12)}{32-6} = \frac{13}{26} = 1/2$

$m_{DG} = \frac{y_2-y_1}{x_2-x_1} = \frac{-12-2}{6-(-8)} = \frac{-14}{14} = -1$

Two lines have equal slopes which means they are parallel and the other 2 lines are not parallel making this a trapezoid.

22. A(-5,-5) B(5,-7) C(8,8)

$L_{AB}^2 = (x_2-x_1)^2+(y_2-y_1)^2 = [5-(-5)]^2+[-7-(-5)]^2 = (10)^2+(-2)^2 = 100+4$

$L_{AB} = \sqrt{104}$ units

$L_{BC}^2 = (x_2-x_1)^2+(y_2-y_1)^2 = [18-5]^2+[8-(-7)]^2 = (13)^2+(15)^2 = 169+225$

$L_{BC} = \sqrt{394}$ units

$L_{AC}^2 = (x_2-x_1)^2+(y_2-y_1)^2 = [18-(-5)]^2+[8-(-5)]^2 = (13)^2+(13)^2 = 169+169$

$L_{AC} = \sqrt{338}$ units

$\because L_{AB} \neq L_{BC} \neq L_{AC}$

$\therefore$ It is a scalene triangle.

23. D(-1,2) E(-4,1) F(2,-2) G(1,1) trapezoid.

$m_{DE} = \frac{y_2-y_1}{x_2-x_1} = \frac{2-1}{-1-(-4)} = \frac{1}{3}$

$m_{EF} = \frac{y_2-y_1}{x_2-x_1} = \frac{-2-1}{2-(-4)} = \frac{-3}{6} = -\frac{1}{2}$

$m_{FG} = \frac{y_2-y_1}{x_2-x_1} = \frac{-2-1}{2-1} = \frac{-3}{1} = -3$

$m_{DG} = \frac{y_2-y_1}{x_2-x_1} = \frac{1-2}{1-(-1)} = \frac{-1}{2}$

$\because m_{EF} = m_{DG}$, the lines are parallel and $m_{DE} \neq m_{DG}$

$\therefore$ It is a trapezoid