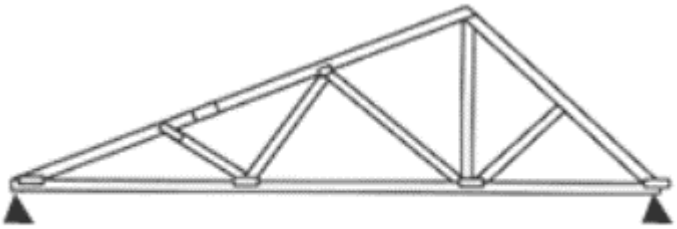
The Cosine Law

When a triangle does not have a right angle, and we are unable to apply the sine law, it is possible to draw an altitude that divides a known side length into two parts.

Example – A cottage is being designed with an asymmetrical roof in order to install solar panels on one face. The rafters on one side will be 18 ft long and sloped at 40^{o}. The roof truss will be 21 ft long.

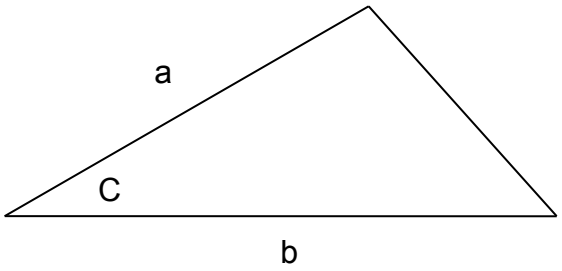
a) Determine the height of the roof.
b) Determine the length of the shorter rafters.



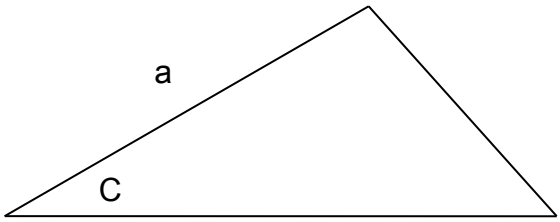
The cosine law is a formula for calculating an opposite angle or length in any triangle when we do not know a full ratio (an angle and its opposite side length).

We can prove the cosine law by drawing an altitude in the second diagram below to illustrate that side a, angle C and side b create both:

an acute triangle with side c



a right angle triangle with hypotenuse c



We must express h, b_1 and b_2 in terms of a, b and C:

Then we can solve for c using the Pythagorean theorem:

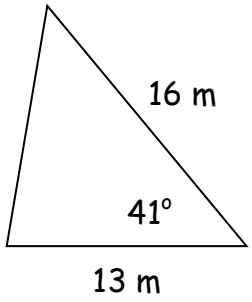
We can apply the Cosine Law if we know an angle between two side lengths.

After we apply the cosine law, we will have enough information to solve for one of the missing angles using the sine law and the third using angle sum triangle theorem.

However, we should be aware that:

- the largest angle is opposite to the largest side
- the largest angle could be obtuse (greater than 90 degrees)
- the sine law can only provide an acute angle (less than 90 degrees)

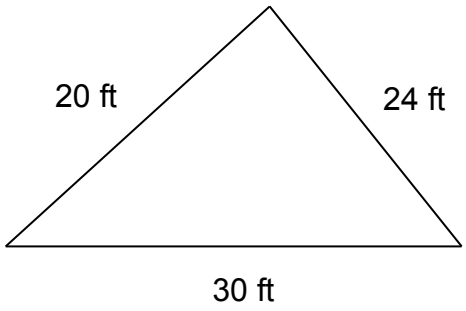
Example – Solve the following acute triangle.



We can rearrange the Cosine Law in order to solve for any angle in a triangle if all three side lengths are known.

* If we find the largest angle using cosine law then sine law can effectively be used to find the remaining two angles.

Example – Solve for the following acute triangle.



Homework – Please complete: questions #1, 3 and 7 on page 409
questions #5 and 9 on page 418