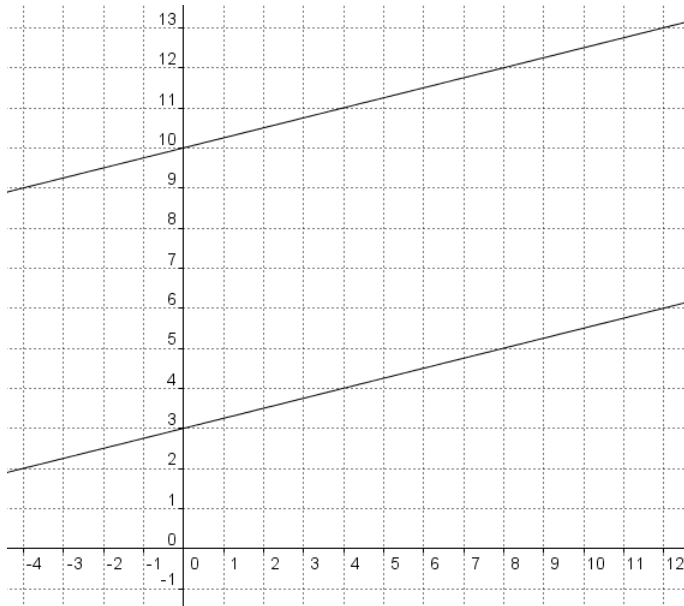


## The Shortest Distance From a Point to a Line

We can use a graph to illustrate the information and steps required to determine the shortest distance between two parallel lines:



Since graphing is not always an efficient or accurate method, we must identify the most important step in this process and how we can accomplish it using algebra:

**Graphing:** Choose a starting point on one of the lines then draw the shortest distance line segment until it intersects a point on the other line.

**Algebra:** Write an equation for a line that passes through the starting point on one of the lines and is perpendicular to the other line then solve a linear system to determine where the perpendicular lines intersect.

We can summarize the entire algebraic process in five steps:

### Steps for Determining the Shortest Distance From a Point to a Line

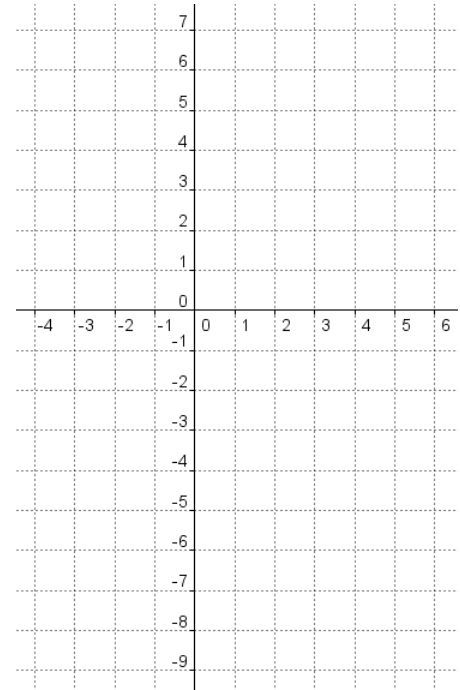
1. Determine the slope of the given line (rearrange to  $y = mx + b$  if needed).
2. Determine the slope of the perpendicular line (negative reciprocal of  $m$ ).
3. Determine the equation for the line through the given point with slope  $m_{\perp}$ .
4. Determine the point of intersection of  $y = mx + b$  and  $y = m_{\perp}x + b$ .
5. Determine the distance from the given point to the point of intersection.

Example – Determine the shortest distance from the point P  $(-1, 5)$  on  $x - 3y = -16$   
to the line  $y = \frac{1}{3}x - 8$ .

Step 1 – Slope of the given line:  $m =$

Step 2 – Negative reciprocal slope:  $m_{\perp} =$

Step 3 – Determine the equation for the line  
through point P with slope  $m_{\perp}$ :



Step 4 – Determine the point of intersection of  $y = mx + b$  and  $y = m_{\perp}x + b$ .

Step 5 – Determine the distance from the given point to the point of intersection

When we ask for the shortest distance between two parallel lines the answer will be the same no matter which point we choose on one of the lines.

Therefore, we are really just asking for the shortest distance from a point to a line.

Example – Determine the shortest distance from P  $(-19, -11)$  to  $5x + 3y = 42$ .

Homework: Please solve the following problems.

1. Determine the shortest distance from P (14, 17) on  $x + 4y = 82$  to the parallel line

$$y = -\frac{1}{4}x - 5.$$

2. Determine the shortest distance from P  $(-7, 21)$  to the line  $2x - 5y = 55$ .

3. Determine the shortest distance from P (8, 9) to the line  $3x + 4y = -40$ .

Answers:

1. The shortest distance is  $\sqrt{612}$  (approx. 24.74) units from (14, 17) to (8, -7).
2. The shortest distance is  $\sqrt{1044}$  (approx. 32.311) units from (-7, 21) to (5, -9).
3. The shortest distance is 20 units from (8, 9) to (-4, -7).