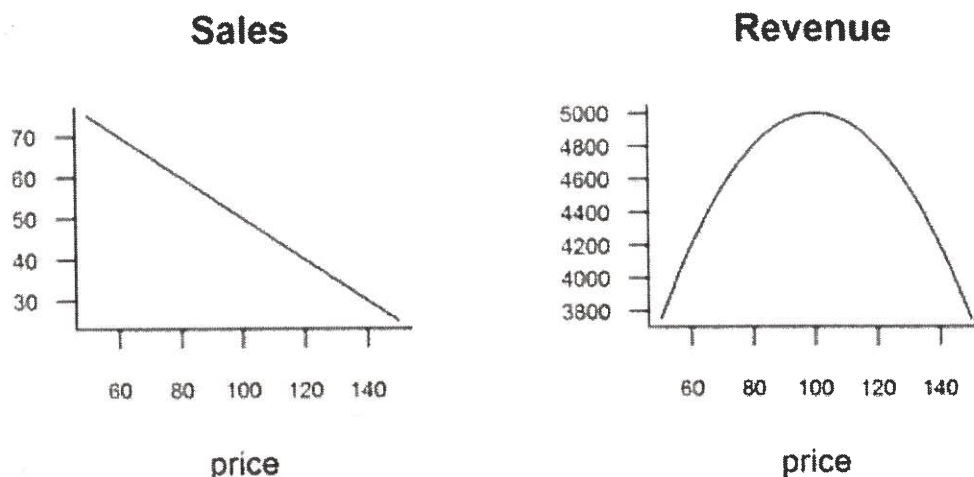


Solving Revenue Problems

Business owners often use revenue equations to determine how to maximize their revenue. Revenue is calculated by multiplying the number of items sold by the price of the item. The more a business charges its customers, the more it earns. However if the business charges too much money, its sales will decrease and they will lose revenue.



A graph of revenue versus price has a parabolic shape. This means that the maximum revenue earned is equivalent to the vertex of the corresponding quadratic relation.

As we know, it is possible to determine the vertex based on the x-intercepts.

General Form of a Revenue Equation:

$$\text{Revenue} = (\# \text{ of items})(\text{price per item})$$

Example 1 - A theatre company has 300 season ticket subscribers. The manager has decided to raise the price of a season ticket from the current price of \$400. A survey of subscribers has determined, that for every \$20 increase, 10 subscribers would not renew their season tickets.

- a. Determine an expression for the company's revenue.

let x represent the # of price increases.

$$R = (\text{\# of subscribers})(\text{price/subscriber})$$

$$R = (300 - 10x)(400 + 20x)$$

- b. Determine the number of price changes the company can make to maximize revenue.

$$R = (300 - 10x)(400 + 20x)$$

$$\begin{aligned} 300 - 10x &= 0 \\ \frac{-10x}{-10} &= \frac{-300}{-10} \end{aligned}$$

$$x = 30$$

$$\begin{aligned} 400 + 20x &= 0 \\ \frac{20x}{20} &= \frac{-400}{20} \end{aligned}$$

$$x = -20$$

$$x = \frac{30 - 20}{2}$$

$$= \frac{10}{2}$$

$$= 5$$

$\therefore 5$ price changes

- c. Determine the maximum revenue.

$$R = (300 - 10x)(400 + 20x)$$

$$= [300 - 10(5)][400 + 20(5)]$$

$$= (300 - 50)(400 + 100)$$

$$= 250(500)$$

$$= 125,000$$

\therefore max revenue is \$125,000.

- d. Determine the number of ^{sub}tickets sold to achieve maximum revenue.

$$\text{\# subscribers} = 300 - 10x$$

$$= 300 - 10(5)$$

$$= 250$$

- e. Determine the ticket price that maximizes the revenue.

$$\text{price} = 400 + 20x$$

$$= 400 + 20(5)$$

$$= \$500 \text{ per subscriber}$$

Example 2 - If a farmer harvests his crop today, he will have 1200 bushels worth \$2 per bushel. Every week that he waits, the crop increases by 100 bushels, but the price drops by 10 cents per bushel.

- a. Write a revenue equation to represent this scenario.

let x be the # of weeks the farmer should wait.

$$R = (\text{\# of bushels})(\text{price / bushel})$$

$$R = (1200 + 100x)(2 - 0.1x)$$

- b. When should he harvest his crop to maximize his revenue?

$$R = (1200 + 100x)(2 - 0.1x)$$

$$1200 + 100x = 0$$

$$\frac{100x}{100} = \frac{-1200}{100}$$

$$x_1 = -12$$

$$2 - 0.1x = 0$$

$$\frac{-0.1x}{-0.1} = \frac{-2}{-0.1}$$

$$x = 20$$

$$x = \frac{-12 + 20}{2}$$

$$= \frac{8}{2}$$

$$= 4$$

\therefore He should wait 4 weeks.

- c. What is the maximum revenue?

$$R = (1200 + 100x)(2 - 0.1x)$$

$$= [1200 + 100(4)][2 - 0.1(4)]$$

$$= (1200 + 400)(2 - 0.4)$$

$$= 1600(1.6)$$

$$= 2560$$

\therefore max revenue is \$2560.

- d. How many bushels will he have to sell to achieve maximum revenue?

$$\text{\# bushels} = 1200 + 100x$$

$$= 1200 + 100(4)$$

$$= 1600 \text{ bushels}$$

- e. What price per bushel will achieve the maximum revenue?

$$\text{price} = 2 - 0.1x$$

$$= 2 - 0.1(4)$$

$$= \$1.60 / \text{bushel}$$

Example 3 - A store has been selling 200 digital receivers a week for \$350 each. A market survey indicates that for each \$10 rebate offered to buyers, the number of receivers sold will increase by 20 each week.

- a. Determine the revenue equation for the store.

let x be the # of rebates offered.

$$R = (\# \text{ of D.R.}) (\text{price / D.R.})$$

$$R = (200 + 20x)(350 - 10x)$$

- b. How large a rebate should be offered to maximize revenue?

$$R = (200 + 20x)(350 - 10x)$$

$$200 + 20x = 0$$

$$\frac{20x}{20} = \frac{-200}{20}$$

$$x = -10$$

$$350 - 10x = 0$$

$$\frac{-10x}{-10} = \frac{-350}{-10}$$

$$x = 35$$

$$x = \frac{-10 + 35}{2}$$

$$= -\frac{25}{2}$$

$$= 12.5$$

\therefore 12.5 rebates
or a \$125
rebate
should be
offered.

Summary

$$12.5(10) = \$125$$

To determine the number of price changes or the number of weeks to wait...

Determine the axis of symmetry.

$$x = \frac{r+s}{2}$$

To determine the maximum revenue...

Substitute x into revenue eqⁿ and solve for y .

To determine the selling price that earns maximum revenue...

Substitute x into price portion of revenue eqⁿ

To determine the number of items sold that earns maximum revenue...

Substitute x into # of items portion of
revenue eqⁿ

Homework

1. Answer question 16 on page 272 of the textbook.
↓
abc only
2. The Environmental Club sells sweatshirts as a fundraiser. They sell 1200 shirts a year at \$20 each, but they are planning to increase the price. A survey indicates that for every \$2 increase in price, there will be a drop of 60 sales a year.
 - a. What is the revenue equation that the Environmental Club will use?
 - b. What should the selling price be in order to maximize the revenue?
 - c. How much money will the Environmental Club earn?
3. If 400 people attend the student theatre production when the admission price is \$3.50, and if the attendance decreases by 20 people for each 25 cents added to the price.
 - a. What is the revenue equation that represents this scenario?
 - b. How many price increases should the group make to maximize revenue?
 - c. What admission price will yield the greatest revenue?
 - d. How many people will attend the show?
4. An electrical appliance retailer finds that he can sell 200 espresso machines each month at a price of \$500 for each machine. For each reduction of \$20, he sells 10 more machines each month.
 - a. What selling price will produce the maximum revenue per month?
 - b. How many machines will the retailer sell per month?
 - c. What is the maximum revenue earned by the retailer per month?
5. Right now George has 600 pumpkins and he could sell them all at 50 cents each. If he waits, he can increase the price by 10 cents every week until he is ready to sell. However, 50 pumpkins will rot each week.
 - a. How long should he wait before selling his pumpkins in order to earn the maximum revenue from his crop?
 - b. What is the maximum revenue that he will earn?