

## Solving Quadratic Equations

To solve a quadratic equation means to:

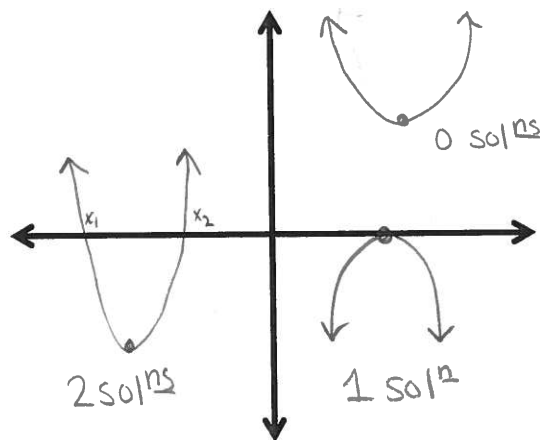
determine the x-intercepts.

There are three possible types of answers.

A. 2 solutions

B. 1 solution ( $x_1 = x_2$ )

C. 0 solutions



Today we will focus on three strategies for solving quadratic equations in vertex form.

1. Factor the equation to determine the x-intercepts.  $y = a(x-r)(x-s) \rightarrow x_1 = r \quad x_2 = s$
2. Use the quadratic formula to determine the x-intercepts.  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
3. Use opposite operations to determine the x-intercepts. (SAMPERS)  
\* this only works if the quadratic relation has only one term with x.

Before solving a quadratic equation by factoring or with the quadratic formula, ensure it is in standard form,  $ax^2 + bx + c = 0$ . You may have to simplify or rearrange the equation first.

To solve a quadratic equation including fractions, eliminate the fractions by multiplying each term of the equation by the lowest common denominator. Then rearrange the equation and solve.

1. Solve each of the quadratic equations below.

1. rearrange

$$\text{a. } 4x^2 = 20x$$

$-20x \quad -20x$

$$\text{b. } 9x^2 - 36 = 0$$

$+36 \quad +36$

\* use opposite operations.

2. factor

$$4x^2 - 20x = 0$$

$$\frac{9x^2}{9} = \frac{36}{9}$$

$$4x(x-5) = 0$$

$$x^2 = 36$$

3. determine intercepts

$$\frac{4x}{4} = \frac{0}{4}$$

$$\frac{x-5}{+5} = \frac{0}{+5}$$

$$x = 0$$

$$x = 5$$

$$\sqrt{x^2} = \sqrt{36}$$

$$x = \pm 6$$

$$\boxed{x_1 = 0 \quad x_2 = 5}$$

$$\boxed{x_1 = 6 \quad x_2 = -6}$$

1. rearrange

$$\text{c. } 2x^2 + 7 = -15x$$

$+15x \quad +15x$

$$2x^2 + 15x + 7 = 0$$

2. factor using product-sum

$$(2x+1)(x+7) = 0$$

$$P = 14 \quad S = 15$$

$$14, 1$$

$$2x+1 = 0$$

$$x+7 = 0$$

3. determine intercepts

$$\frac{2x}{2} = \frac{-1}{2}$$

$$x = -7$$

$$x = -\frac{1}{2}$$

$$\boxed{x_1 = -\frac{1}{2} \quad x_2 = -7}$$

2. Simplify and then solve the quadratic equations below.

$$\frac{9x^2}{4} + 3x - \frac{7}{2} = 0$$

\* the common denominator between 4 and 2 is 4, so multiply every term by 4.

1. multiply  
by 4

$$4\left(\frac{9x^2}{4}\right) + 4(3x) + 4\left(-\frac{7}{2}\right) = 4(0)$$

2. simplify

$$\frac{36x^2}{4} + 12x - \frac{28}{2} = 0$$

$$9x^2 + 12x - 14 = 0$$

$$P = -126 \quad S = 12$$

\* easier to use quad.  
formula than factor

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-12 \pm \sqrt{(12)^2 - 4(9)(-14)}}{2(9)}$$

$$= \frac{-12 \pm \sqrt{144 + 504}}{18}$$

$$X_1 = \frac{-12 + \sqrt{648}}{18}$$

$$X_2 = \frac{-12 - \sqrt{648}}{18}$$

$$X_1 = 0.75$$

$$X_2 = -2.08$$

## Solving Quadratic Equations - Homework Worksheet

1. Solve each of the following quadratic equations by factoring.

a.  $x^2 + 3x = 0$

b.  $x^2 - 22 = -9x$

c.  $4x^2 - 12x = -9$

d.  $6x^2 + 9 = -15x$

e.  $-x^2 - 10x - 16 = 0$

2. Solve each of the following quadratic equations by the quadratic formula.

a.  $7x^2 - 12x = 9$

b.  $4x^2 = 2.8x + 4.8$

c.  $10x^2 - 7 = 45x$

3. Simplify and solve each of the following equations.

a.  $\frac{x^2}{2} + x + \frac{1}{2} = 0$

b.  $\frac{x^2}{4} - \frac{x}{3} = \frac{1}{3}$

c.  $\frac{x^2 + 11}{10} = 2$

d.  $3x^2 - 7 = 30 - 12x^2$

### Answer Key

1a)  $x_1 = 0$   
 $x_2 = -3$

1b)  $x_1 = -11$   
 $x_2 = 2$

1c)  $x = \frac{3}{2}$

1d)  $x_1 = -\frac{3}{2}$   
 $x_2 = -1$

1e)  $x_1 = -8$   
 $x_2 = -2$

2a)  $x_1 \approx 2.28$   
 $x_2 \approx -0.56$

2b)  $x_1 = 1.5$   
 $x_2 = -0.8$

2c)  $x_1 \approx 4.65$   
 $x_2 \approx -0.15$

3a)  $x = -1$

3b)  $x_1 = -\frac{2}{3}$   
 $x_2 = 2$

3c)  $x_1 = 3$   
 $x_2 = -3$

3d)  $x_1 \approx 1.57$   
 $x_2 \approx -1.57$