Projectile Motion in Standard Form

Projectile motion can be modeled in:

$y = a(x-h)^2 + k$ vertex form

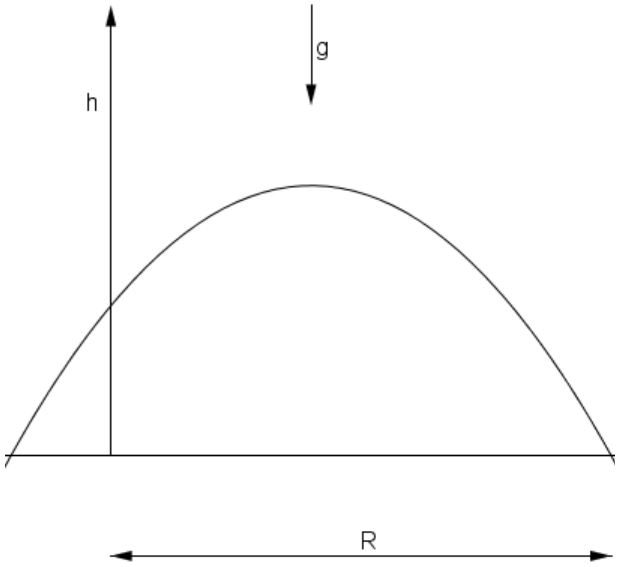
maximum height when $x =$

$y = a(x-r)(x-s)$ factored form

maximum height when $x =$

$y = ax^2 + bx + c$ standard form

maximum height when $x =$



We can calculate the height at any location by substituting x into the equation.

Example – An astronaut hits a golf ball from the edge of a crater on the moon. The height, h (in metres), of the ball above the bottom of the crater depends on the time, t (in seconds), after the ball was struck. The height can be modeled by $h = -0.8t^2 + 12t + 19$.

a) Determine the initial height of the ball.

b) Determine the maximum height of the ball.

c) Determine the height of the ball 14 seconds after it was struck.

When the height is known we can solve for x using either:

→ opposite operations (when the variable appears only once in the equation)

→ factoring (which is the opposite operation of expanding)

→ quadratic formula (which has opposite operations built into it)

Example – The height, h (in metres), of a football above the ground depends on how far it has travelled horizontally, d (in metres), from where it was thrown. The height can be modeled by $h = -0.2d^2 + 1.6d + 1.8$.

a) Determine where the football hits the ground.
b) Determine where the football reaches a height of 4.3 m.

Remember that the quadratic formula accomplishes two major tasks:

→ completing the square to convert from standard form to vertex form
→ opposite operations to solve for x when y is equal to zero

Therefore, when a quadratic relation is given in vertex form:

→ we do not need to use the quadratic formula (half of the work is done already)
→ we cannot use the quadratic formula (unless we convert back to standard form)

Example – The height, h (in metres), of a snowboarder above the ground depends on the time, t (in seconds), after the snowboarder leaves the edge of a jump. The height can be modeled by $h = -4.9(t - 0.8)^2 + 5.6$. Determine the initial height, maximum height and flight time of the snowboarder. Sketch the trajectory and label the coordinates of the important points.

Homework – Please complete the questions below and # 6, 7 and 11 on page 301.

1. The height, h (in metres), of a snowboarder above the ground depends on how far the snowboarder has travelled horizontally, d (in metres), from the edge of a jump. The height can be modeled by $h = -0.1d^2 + 0.8d + 2.5$.

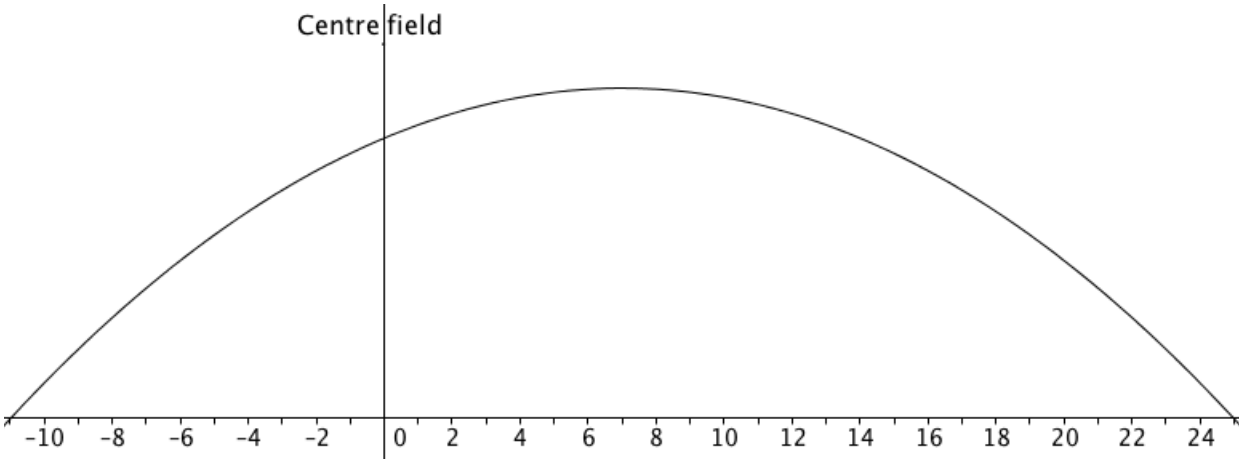
   a) Determine how far the snowboarder travelled horizontally before landing.
   b) Determine where the snowboarder was more than 4 m above the ground.

2. The height, h (in metres), of a golf ball above the fairway depends on the time, t (in seconds), after the ball was struck from an elevated tee position. The height can be modeled by $h = -4.9(t - 1.5)^2 + 16.6$.

   a) What was the initial height of the ball?
   b) What was the maximum height of the ball?
   c) When did the ball hit the ground?
   d) Sketch the trajectory of the ball and label the coordinates of important points.

3. The height, h (in metres), of a soccer ball above the ground can be modeled by $h = -0.03(d + 11)(d - 25)$ where d (in metres) represents how far the ball is horizontally from the centre line of the soccer field.

   a) Determine the maximum height of the soccer ball above the ground.
   b) Determine the height of the ball when it is directly above the centre line.
   c) Determine where the ball reaches a height of 6.6 m above the ground.



Answers:

1. a) 10.40 m b) 3 m to 5 m

2. a) 5.57 m b) 16.6 m c) 3.34 s d) sketch

3. a) 9.72 m b) 8.25 m c) 3.2 m before and 17.2 m after centre field

Answers:

1. a) The snowboarder travelled 10.40 m horizontally before landing.

   b) The snowboarder reached a height of 4 m above the ground at a horizontal distance of 3 m from the jump and 5 m from the jump. Between these two locations the snowboarder was more than 4 m above the ground.

2. a) The initial height was 5.57 m.

   b) The maximum height was 16.6 m.

   c) The ball hits the ground after 3.34 s.

   d) The initial height is at the point (0, 5.57), the maximum height is at the point (1.5, 16.6) and the ball hits the ground at the point (3.34, 0).

3. a) The maximum height of the ball is 9.72 m above the ground.

   b) The height of the ball above the centre line of the soccer field is 8.25 m.

   c) The ball reaches a height of 6.6 m when it is 3.2 m before the centre of the field and when it is 17.2 m after the centre of the field.