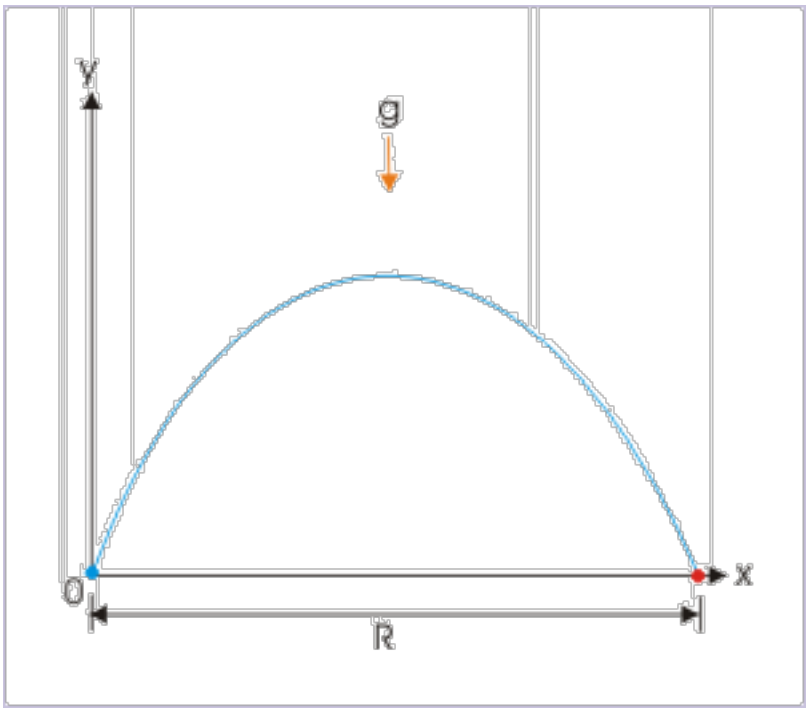
Projectile Motion

Projectile motion refers to the path (or trajectory) that an object follows from launch to impact. If the only force acting on the object is the force of gravity, then this path is parabolic (it can be modeled by a quadratic relation).

The path of a projectile will be completely symmetrical if it is launched from the same height at which it lands. Otherwise, the parabola will be "cut short" on one end if its height at launch is different from its height at impact.

The force of gravity causes the object to slow down as it rises and speed up as it falls. The velocity of the object is momentarily horizontal when it reaches its maximum height (or vertex).

The horizontal displacement of the object from launch to impact is called the range of the projectile. This can be confusing since we usually use the term "range" to refer to the y–values of a relation.



If the projectile motion of an object is represented by the equation $y = a(x-h)^2 + k$ then the maximum height of the object is _____ and it occurs when _____.

* We can use order of operations to solve for y when we know the location, x.

* We can use opposite operations to solve for x when we know the height, y.

Example – The height, h (in metres), of a football t seconds after it was kicked was modeled by the following quadratic relation:

$$h = -4.9(t - 3)^2 + 45$$

a) What was the maximum height of the football?

b) At what time did the football reach its maximum height?

c) What was the initial height of the football?

d) Sketch the trajectory of the football. Label the y–intercept and vertex.

e) What was the height of the football after 2 seconds?

f) When did the football reach a height of 30 metres?

g) How long did it take for the football to hit the ground?

Homework

1. The height, h (in metres), of a football t seconds after it was thrown was modeled by the following quadratic relation:

$$h = -4.9(t-2)^2 + 22$$

   a) What was the initial height of the ball?
   b) What was the maximum height of the ball?
   c) What was the height of the ball after 2.5 s after it was thrown?
   d) When did the ball reach a height of 18 m?
   e) How long did it take for the ball to hit the ground?

2. A football is kicked at an angle of 30° to the ground, at an initial speed of 19.6 m/s, from a height of 1 m. Two quadratic relations can be used to model its height, h (in metres), above the ground.

   With respect to time, t (in seconds), the height is given by:

$$h = -4.9(t-1)^2 + 5.9$$

   With respect to the horizontal distance, d (in metres), the height is given by:

$$h = -0.017(d-17)^2 + 5.9$$

   a) What is the maximum height of the football?
   b) When does the maximum height occur?
   c) Where does the maximum height occur?
   d) When does the football reach a height of 3.5 m?
   e) Where does the football reach a height of 3.5 m?

3. The following quadratic relation gives the height, h (in metres), of a baseball depending on the horizontal distance, d (in metres), it has travelled since being hit with an initial speed of 27 m/s at an angle of 88° above the ground:

$$h = -6(d-2.5)^2 + 39$$

   a) What was the maximum height of the ball?
   b) What was the height of the ball when it was hit?
   c) How far did the ball travel horizontally before it hit the ground?
   d) At what horizontal distance had the ball reached a height of 29 m?

4. Answer questions #12 and 14 on page 186 of the textbook.