Power Laws

A power is a product of identical factors. $\text{base} \rightarrow \underbrace{2^{\overset{\text{exponent}}{4}}}_{\text{power}} = 2 \cdot 2 \cdot 2 \cdot 2$

A power can be written in exponential form or expanded form.

Exponential Form → 3^5

Expanded Form → $(3)(3)(3)(3)(3)$

Negative Signs and Bases

If a negative sign is in brackets, it is part of the base and the exponent must be applied to it.
$(-3)^2 = (-3)(-3)$
$(-3)^2 = 9$

If a negative sign is not in brackets, it is not part of the base and the exponent does not apply to it.
$-3^3 = -(3)(3)$
$-3^3 = -9$

Power Law Name	**Power Law**	**Explanation and Examples**
The Product Law	When multiplying powers with the same base, add the exponents. $x^a \cdot x^b = x^{a+b}$	$2^2 \cdot 2^4$ $= (2)(2) \cdot (2)(2)(2)(2)$ $= 2^6$ $(-4)^8(-4)^5$ $= (-4)^{8+5}$ $= (-4)^{13}$ $3^9 \cdot 3^4 \cdot 3$ $= 3^{9+4+1}$ $= 3^{14}$
The Quotient Law	When dividing powers with the same base, subtract the exponents. $x^a \div x^b = x^{a-b}$ $\frac{x^a}{x^b} = x^{a-b}$	$3^4 \div 3^2$ $= \frac{3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3}$ $= 3^2$ $(-7)^{18} \div (-7)^{11}$ $= (-7)^{18-11}$ $= (-7)^7$ $\frac{2^5}{2^3} = 2^{5-3}$ $= 2^2$

Power of a Power

When a power is raised to an exponent, multiply the exponents.

$$(x^a)^b = x^{ab}$$

$$(4^2)^3$$
$$= (4\cdot 4)(4\cdot 4)(4\cdot 4)$$
$$= 4^6$$

$$[(-3)^{12}]^2 = (-3)^{12(2)} = (-3)^{24}$$

$$(2^5)^4 = 2^{5(4)} = 2^{20}$$

Power of a Product Law

An exponent must be applied to each coefficient and variable of a product.

$$(xy)^a = x^a y^a$$

$$(2a^4)^3 = (2)^3 (a^4)^3 = 8a^{12}$$

$$(x^3y^4z)^2 = (x^3)^2 (y^4)^2 (z)^2 = x^6 y^8 z^2$$

Power of a Quotient Law

An exponent must be applied to each coefficient and variable of a quotient.

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$

$$\left(\frac{3x}{4y^2}\right)^2 = \frac{(3)^2(x)^2}{(4)^2(y^2)^2} = \frac{9x^2}{16y^4}$$

Complete the following tables of values.

x	$y=2^x$	
4	16	$\div 2$
3	8	$\div 2$
2	4	$\div 2$
1	2	$\div 2$
0	1	$\div 2$
-1	$0.5 = \frac{1}{2}$	$\div 2$
-2	$0.25 = \frac{1}{4}$	$\div 2$
-3	$0.125 = \frac{1}{8}$	$\div 2$
-4	$0.0625 = \frac{1}{16}$	

x	$y=3^x$	
4	81	
3	27	
2	9	
1	3	
0	1	
-1	$1/3$	$0.\overline{3}$
-2	$1/9$	$0.\overline{1}$
-3	$1/27$	$0.\overline{037}$
-4	$1/81$	0.012345679

The Zero Exponent Law
All powers with an exponent of zero are equal to one. $x^0 = 1$

Negative Exponents
All powers with negative exponents are equal to the reciprocal of the base raised to the equal positive exponent.

$$x^{-a} = \left(\frac{1}{x}\right)^a = \frac{1}{x^a} \qquad \left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a = \frac{y^a}{x^a}$$

a) $\left(\frac{1}{2}\right)^{-2} = \left(\frac{2}{1}\right)^2 = \frac{4}{1} = 4$

b) $\left(\frac{3}{4}\right)^{-3} = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$

c) $(25)^{-1} = \left(\frac{1}{25}\right)^1 = \frac{1}{25}$

Examples – Simplify the following expressions using the power laws. Write all final answers with positive exponents. Evaluate if possible.

a) $x^3 \cdot x^{-8} = x^{3+(-8)} = x^{3-8} = x^{-5} = \left(\frac{1}{x}\right)^5 = \frac{1}{x^5}$

b) $(5xy^2)^3 = (5)^3(x)^3(y^2)^3 = 125x^3y^6$

c) $\frac{y^7}{y^{-2}} = y^{7-(-2)} = y^{7+2} = y^9$

d) $\left(\frac{123}{456789}\right)^0 = 1$

e) $(4xy^2z^3)^{-2}$
$= (4)^{-2}(x)^{-2}(y^2)^{-2}(z^3)^{-2}$
$= 4^{-2}x^{-2}y^{-4}z^{-6}$
$= \left(\frac{1}{4}\right)^2\left(\frac{1}{x}\right)^2\left(\frac{1}{y}\right)^4\left(\frac{1}{z}\right)^6$
$= \frac{1}{16}\cdot\frac{1}{x^2}\cdot\frac{1}{y^4}\cdot\frac{1}{z^6}$
$= \frac{1}{16x^2y^4z^6}$

f) $\frac{1}{x^{-5}}$
$= 1 \div x^{-5}$
$= 1 \div \left(\frac{1}{x}\right)^5$
$= \frac{1}{1} \div \frac{1}{x^5}$
$= \frac{1}{1}\cdot\frac{x^5}{1}$
$= \frac{x^5}{1}$
$= x^5$

g) $z^{13} \cdot z^{-10} \cdot z^{0}$

$= z^{13+(-10)+0}$

$= z^{13-10+0}$

$= z^{3}$

h) $(-5)^{-30} \div (-5)^{-28}$

$= (-5)^{-30-(-28)}$

$= (-5)^{-30+28}$

$= (-5)^{-2}$

$= \left(\frac{1}{-5}\right)^{2}$

$= 1/25$

i) $2^{0} + 3^{0}$

$= 1 + 1$

$= 2$

j) -18^{0}

$= -1$

k) $(a^{3})^{-2}$

$= a^{-6}$

$= \left(\frac{1}{a}\right)^{6}$

$= \frac{1}{a^{6}}$

l) $4^{-2} - 2^{-3}$

$= \left(\frac{1}{4}\right)^{2} - \left(\frac{1}{2}\right)^{3}$

$= \frac{1}{16} - \frac{1}{8}\begin{smallmatrix}\times 2\\ \times 2\end{smallmatrix}$

$= \frac{1}{16} - \frac{2}{16}$

$= -\frac{1}{16}$

m) $\frac{5^{-4}}{2^{-3}}$

$= 5^{-4} \div 2^{-3}$

$= \left(\frac{1}{5}\right)^{4} \div \left(\frac{1}{2}\right)^{3}$

$= \frac{1}{625} \div \frac{1}{8}$

$= \frac{1}{625} \cdot \frac{8}{1}$

$= \frac{8}{625}$