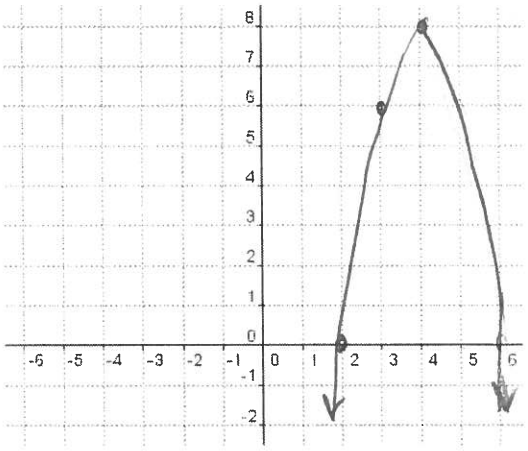
Graphing Quadratic Relations by Factoring

Graph the quadratic relation $y = -2(x-4)^2 + 8$. Write the coordinates of the vertex and x–intercepts.

V(4, -8)

X	Y
1	1(-2) = -2
2	4(-2) = -8
3	9(-2) = -18



The x-intercepts are X=2 and X=6.

Write the quadratic relation $y = -2(x-4)^2 + 8$ in standard form and factored form.

$y = -2(x-4)^2 + 8$
$= -2(x-4)(x-4)+8$
$= -2(x^2 - 4x - 4x + 16) + 8$
$= -2x^2 + 8x + 8x - 32 + 8$
$= -2x^2 + 16x - 24$

$y = -2x^2 + 16x - 24$
$= -2(x^2 + 8x + 12)$
$= -2(x-6)(x-2)$

Conclusion

→ The x-intercepts, 2 and 6, are related to the factors (x-6) and (x-2)

→ The coordinates of each x-intercept makes one factor equal to zero. (x-6 → 6-6 → 0 x-2 → 2-2 → 0)

This is the same as x=h makes the bracket equal zero in vertex form.

→ When x=h the bracket (x-h) equals zero resulting in y=k
When x=r the bracket (x-r) equals zero, resulting in y=0
When x=s the bracket (x-s) equals zero resulting in y=0

This means that we can graph a quadratic relation from standard form, if we are able to factor it.

To graph a parabola we need to know the vertex *(h,k)*. This means we need to determine the axis of symmetry (*x=h*) and the maximum or minimum value (*y=k*).

For the equation $y = a(x-r)(x-s)$, the x-intercepts will be r and s.

(the values of x that make y=0)

The axis of symmetry is directly in between the x-intercepts and can be found by $x = \frac{r+s}{2}$, where x is the x-coordinate of the vertex (*h*).

To find the maximum or minimum value (*k*), we need to find the y coordinate that corresponds to the x-coordinate of the vertex. This can be found by substituting *x* into the original equation. The original equation can be written in standard form ($y = ax^2 + bx + c$) or factored form ($y = a(x-r)(x-s)$).

recall that "a" is the Verhcal stretch/compression and it also determines if the parabola opens up or down.

Once the vertex has been determined, the step pattern can be applied (using *a* from ax^2+bx+c) and the parabola can be completed.

Summary – Graphing a Quadratic Relation from its Factors

1. Find the x-intercepts by factoring
2. Determine the axis of symmetry
3. Determine the maximum or minimum value
4. Determine the vertex
5. Graph

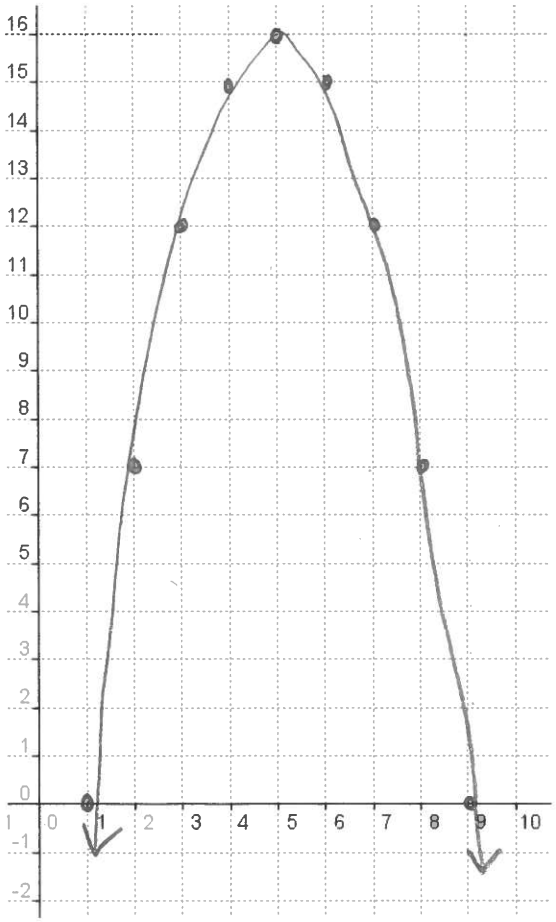
Examples - Graph each of the following quadratic relations by factoring.

a. $y = -x^2 + 10x - 9$

x-intercepts	***Axis of Symmetry***	***Maximum or Minimum Value***	***Vertex***
$0 = -x^2 + 10x - 9$ $0 = -(x^2 - 10x + 9)$ $0 = -(x-9)(x-1)$ $x = 9, 1$	$x = \frac{9+1}{2}$ $= \frac{10}{2}$ $= 5$	$y = -x^2 + 10x - 9$ $= -(5)^2 + 10(5) - 9$ $= -25 + 50 - 9$ $= 16$	$(5, 16)$

$p = 9$
$s = -10$
$-9, -1$

x	y
1	$1(-1) = -1$
2	$4(-1) = -4$
3	$9(-1) = -9$
4	$16(-1) = -16$



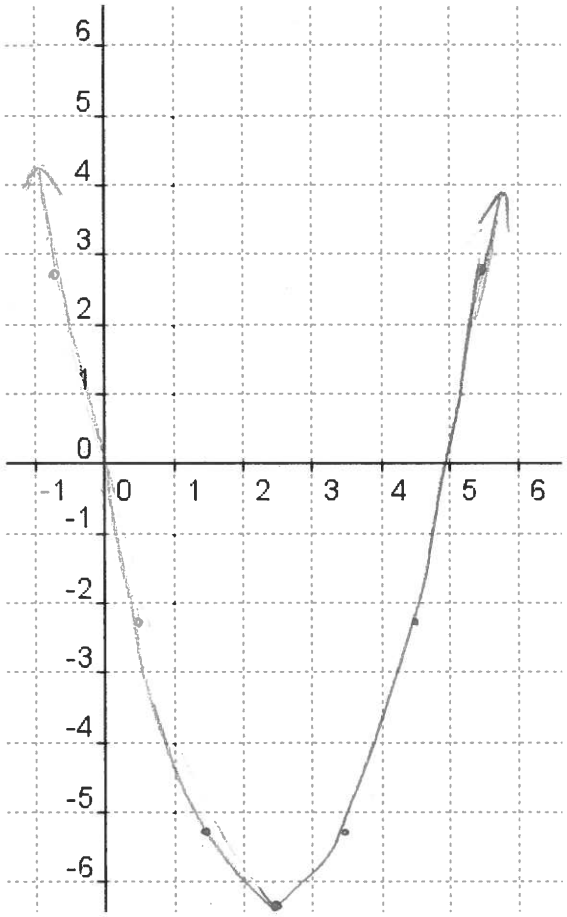
→ $y = -(x-5)^2 + 16$

→ x-int are 1, 9

b. $y = x^2 - 5x$

x-intercepts	***Axis of Symmetry***	***Maximum or Minimum Value***	***Vertex***
$0 = x^2 - 5x$ $0 = x(x-5)$ $x = 0, 5$	$x = \frac{0+5}{2}$ $= \frac{5}{2}$	$y = x^2 - 5x$ $= (\frac{5}{2})^2 - \frac{5}{1}(\frac{5}{2})$ $= \frac{25}{4} - \frac{25}{2}$ $= \frac{25}{4} - \frac{50}{4}$ $= -\frac{25}{4}$	$(\frac{5}{2}, -\frac{25}{4})$ $(2.5, -6.25)$

X	y	
1	1	$-6.25+1 = -5.25$
2	4	$-6.25+4 = -2.25$
3	9	$-6.25+9 = 2.75$



$\Rightarrow y = (x - \frac{5}{2})^2 - \frac{25}{4}$

$\Rightarrow$ x-int are 0 and 5

c. $y = 4x^2 - 8x - 5$

x-intercepts

$0 = 4x^2 - 8x - 5$ $P = -20$, $S = -8$, $-10, 2$

$0 = (2x - 5)(2x + 1)$

$x = \frac{5}{2}, -\frac{1}{2}$

Axis of Symmetry

$x = \frac{\frac{5}{2} - \frac{1}{2}}{2}$

$= \frac{\frac{4}{2}}{2}$

$= \frac{2}{2}$

$= 1$

Maximum or Minimum Value

$y = 4x^2 - 8x - 5$

$= 4(1)^2 - 8(1) - 5$

$= 4(1) - 8 - 5$

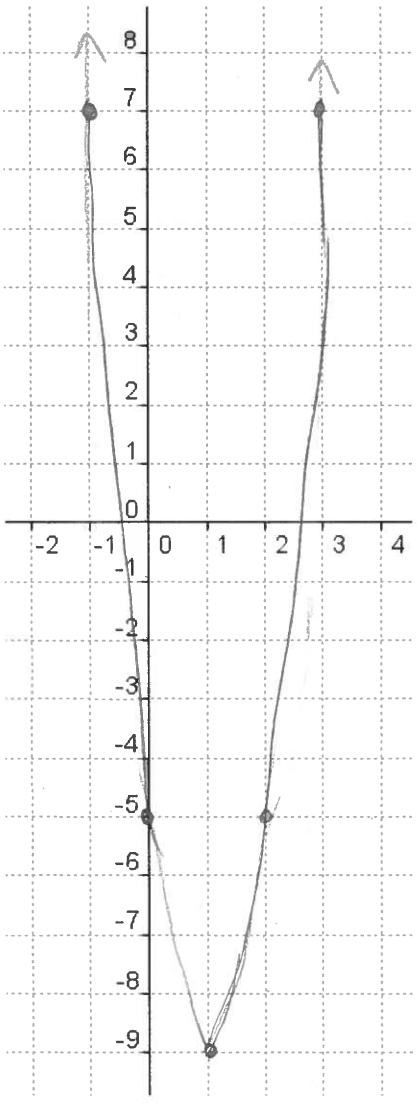
$= 4 - 13$

$= -9$

Vertex

$(1, -9)$

X	Y
1	$1(4) = 4$
2	$4(4) = 16$
3	$9(4) = 36$



→ $y = 4(x - 5/2)^2 - 1/2$

→ x-int are $-1/2$ and $5/2$

Homework

Graph each of the following quadratic relations by determining the x-intercepts, axis of symmetry, maximum or minimum value, and vertex. Show your work.

1. $y = x^2 + 7x + 12$

x-intercepts	*Axis of Symmetry*	*Maximum or Minimum Value*	*Vertex*

2. $y = -x^2 + 8x - 16$

x-intercepts	*Axis of Symmetry*	*Maximum or Minimum Value*	*Vertex*

3. $y = 5x^2 - 20$

x-intercepts	*Axis of Symmetry*	*Maximum or Minimum Value*	*Vertex*

4. $y = 0.5x^2 - 3x - 8$

x-intercepts	*Axis of Symmetry*	*Maximum or Minimum Value*	*Vertex*

5. $y = 2x^2 + 6x + 4$

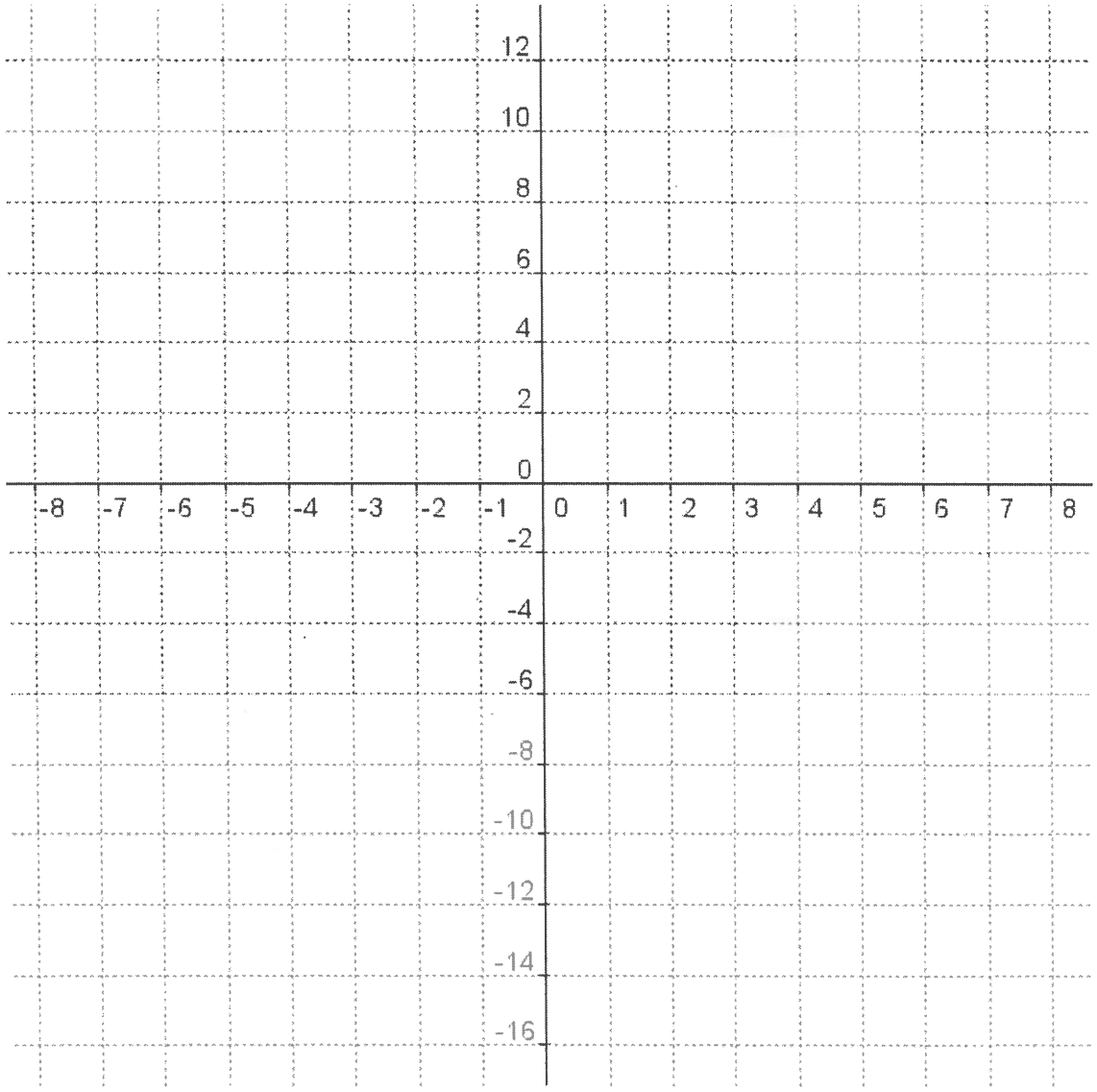
x-intercepts	*Axis of Symmetry*	*Maximum or Minimum Value*	*Vertex*

6. $y = -4x^2 + 4x + 3$

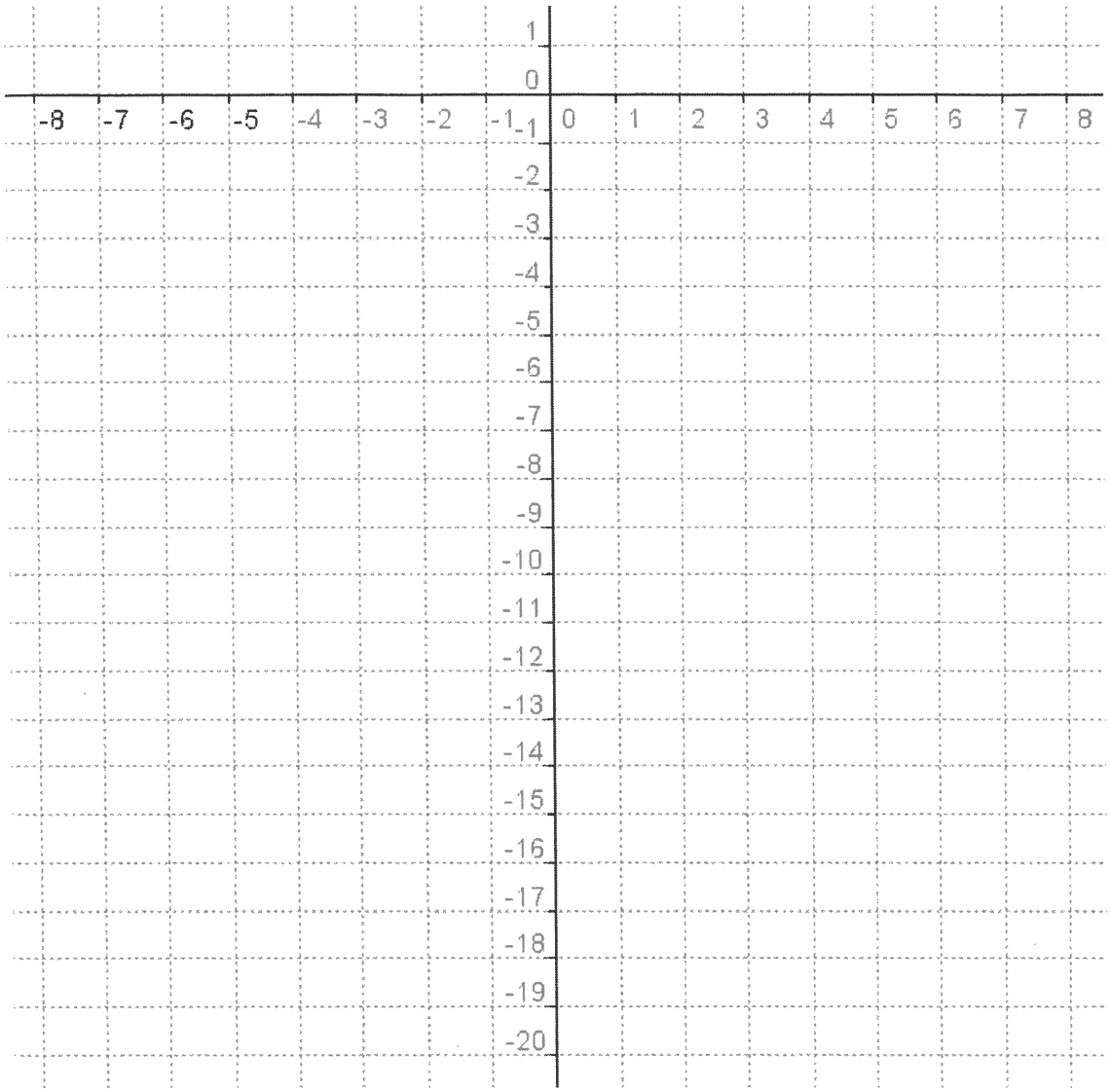
x-intercepts	*Axis of Symmetry*	*Maximum or Minimum Value*	*Vertex*

Homework grids:

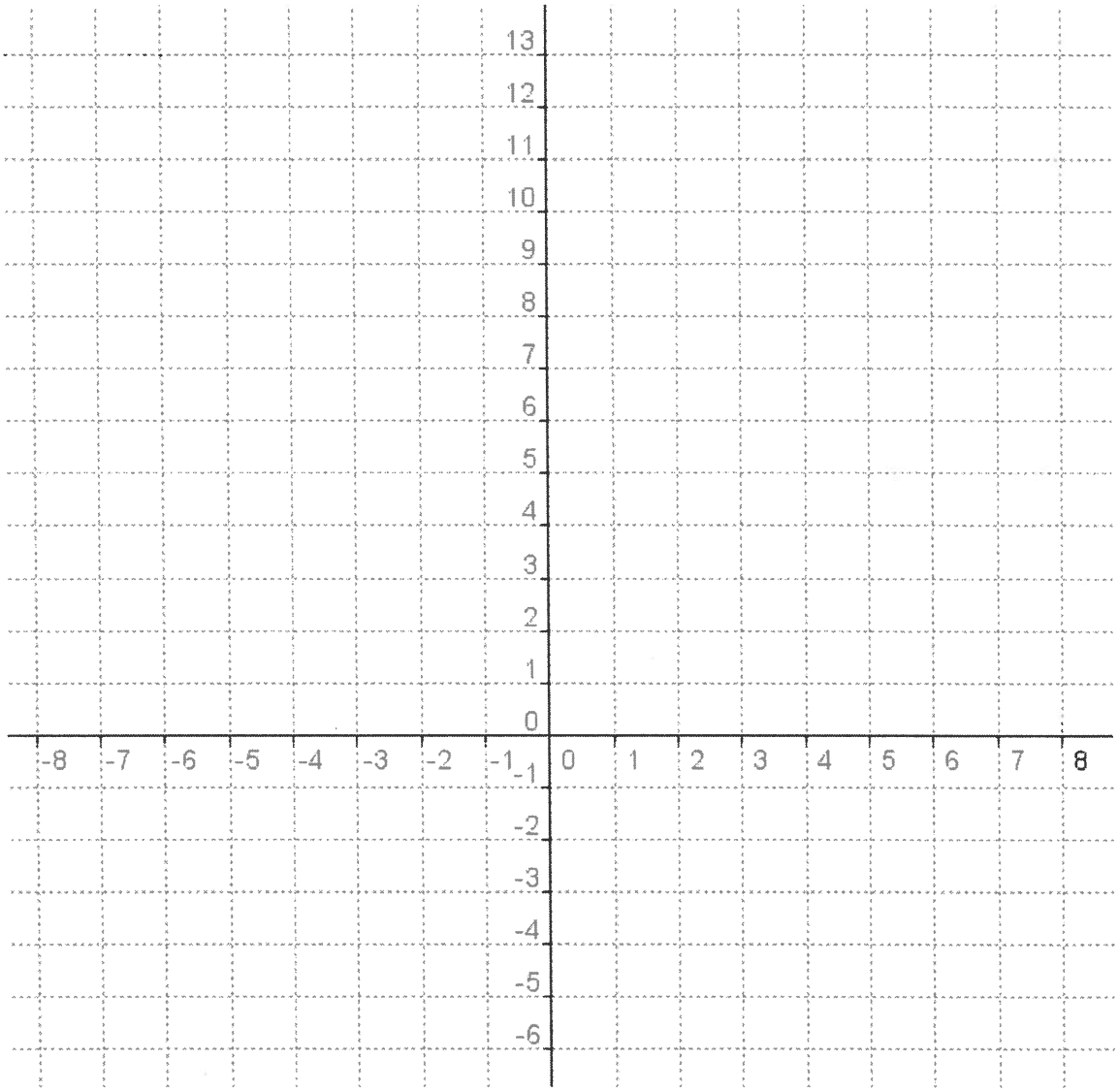
Questions 1 and 2:



Questions 3 and 4:

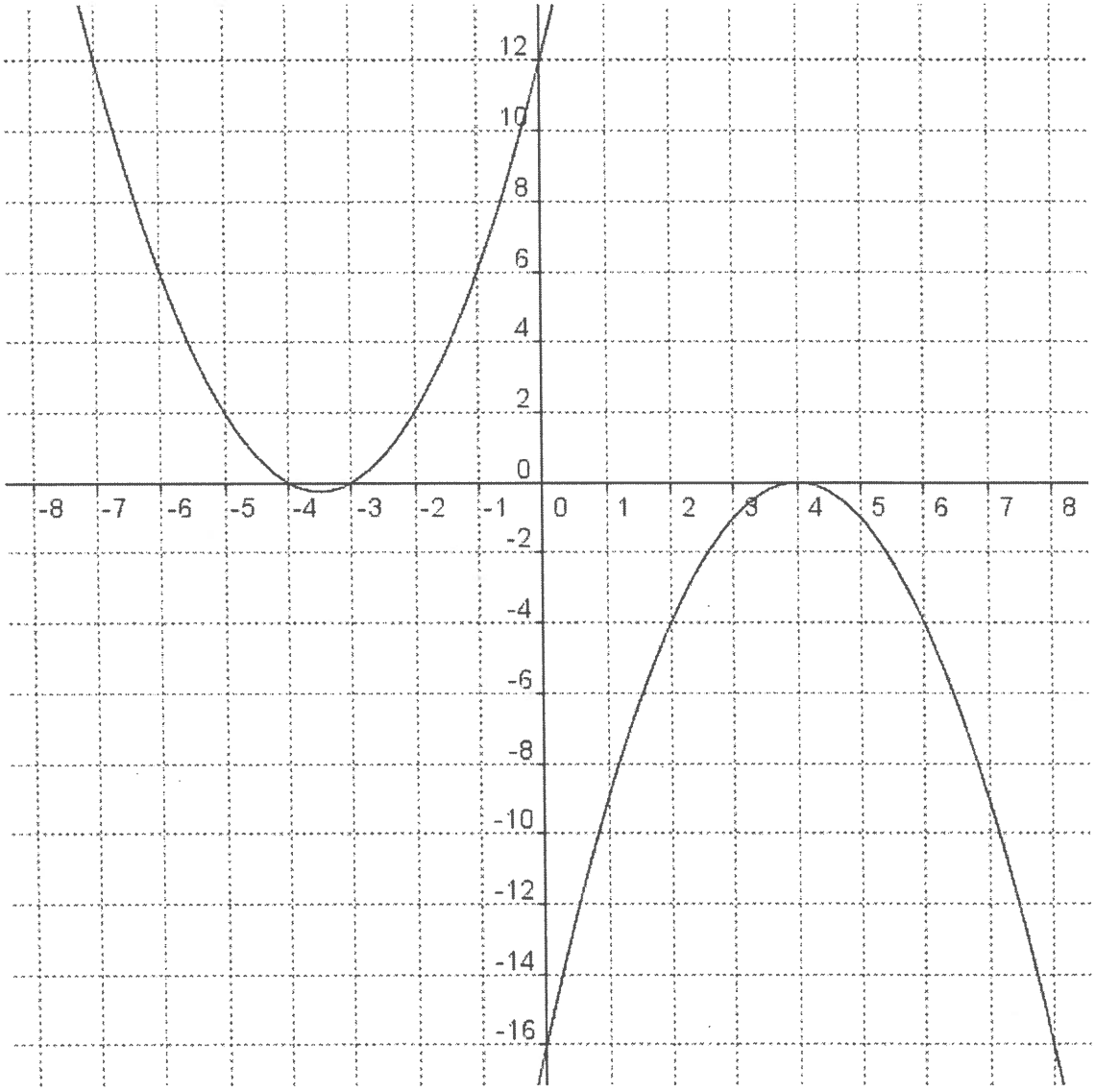


Questions 5 and 6:

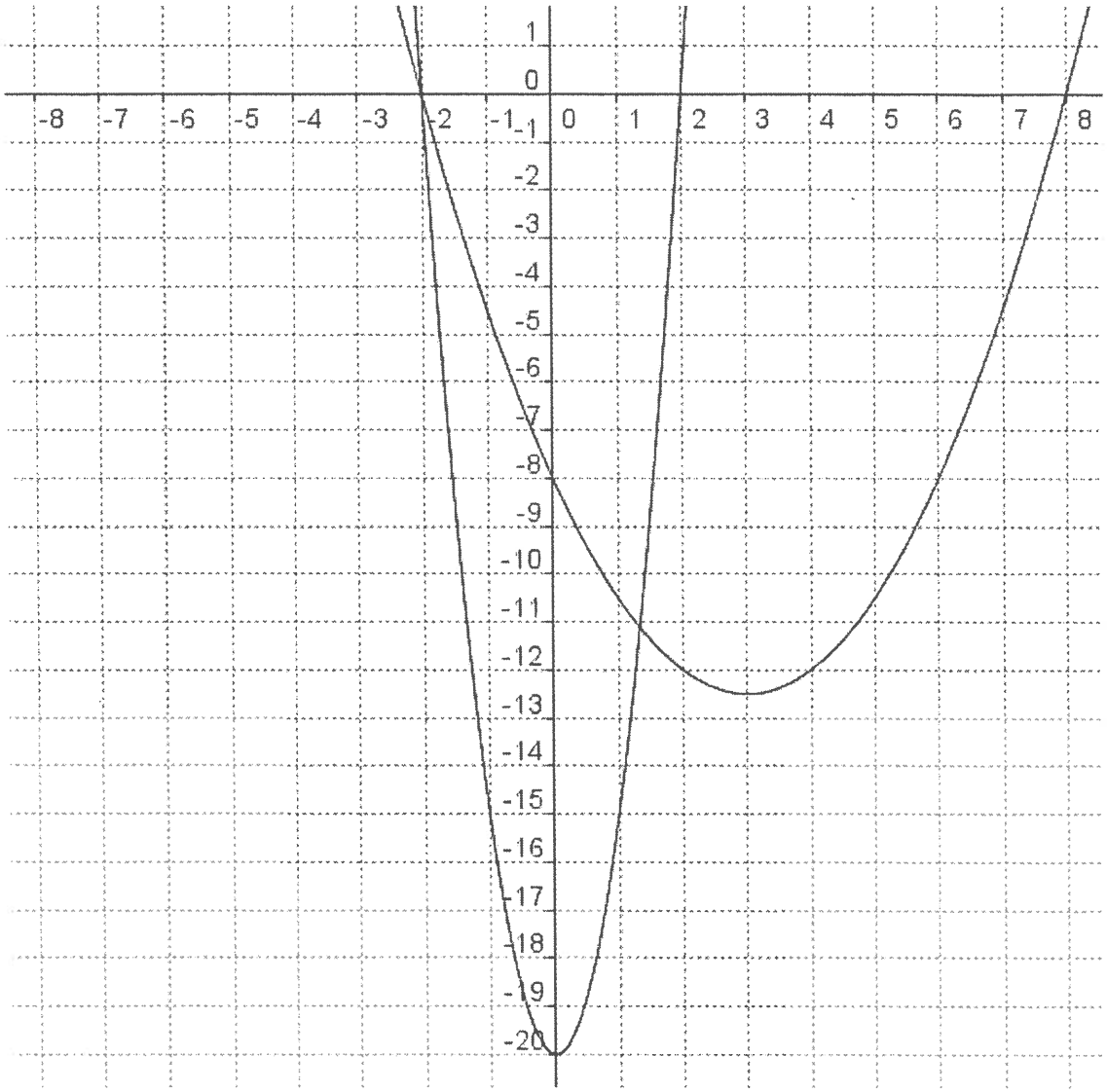


Answers:

Questions 1 and 2:



Questions 3 and 4:



Questions 5 and 6: