Factoring Trinomials by Decomposition

So far, we have been factoring trinomials by writing factors of a and c then expanding and checking the middle term b:

$$\begin{aligned} & ax^2 + b_1x + b_2x + c \\ = \; & ax^2 + bx + c \end{aligned}$$

However, it may be more efficient to determine what the middle coefficients are before we write the factors of a and c.

Let's expand the following expression, and then summarize how the middle coefficients b_1 and b_2 are related to the simplified coefficients a, b and c:

$$\begin{aligned} & (3x-4)(2x+5) \\ = \; & 6x^2 + 15x - 8x - 20 \\ = \; & 6x^2 + 7x - 20 \end{aligned}$$

Summary: $(b_1)(b_2) = (a)(c)$ ← the product

$b_1 + b_2 = b$ ← the sum

Now, we can factor trinomials by decomposing the middle coefficient into b_1 and b_2

Method of decomposition and factoring by inspection

1. Create a table of values to determine two numbers b_1 and b_2 that have the correct product $(a)(c)$ and correct sum (b).
2. Choose factors of a and c that can be expanded to produce b_1 and b_2.

Example – Factor $4x^2 + 5x - 6$

Product $(b_1)(b_2) =$	Sum $b_1 + b_2 =$
(–1)( ) =	–1 + =
(–2)( ) =	–2 + =
(–3)( ) =	–3 + =
(–4)( ) =	–4 + =

(   )(   ) = $4x^2 + 8x - 3x - 6$

Let’s practice factoring by decomposition and inspection. Remember, we want to:

1. Determine the middle terms: $ax^2 + bx + c \quad = \quad ax^2 + b_1x + b_2x + c$

2. Determine factors of a and c: (+) (+) = $ax^2 + b_1x + b_2x + c$

Example – Factor each expression. Include each product and sum table.

$8x^2 + 26x - 15$

$6x^2 - 31x + 18$

We always <u>factor over the integers</u> meaning that we only use integer coefficients in lowest terms. This avoids having too many possibilities or too little information (it avoids working with fractions).

> If there are no values of b_1 and b_2 that have the correct product and sum then a trinomial $ax^2 + bx + c$ cannot be factored over the integers if

Example – Show that $12x^2 - 49x - 30$ cannot be factored over the integers.

(12 rows only including negative sums, could be split into two or three tables)

The **Robin Hood Method** allows us to factor by decomposition and inspection using <u>any</u> two factors of a and the necessary fractions to produce b_1 and b_2 then:

3. Reducing the fractions to lowest terms.
4. Removing common factors and redistributing from the "rich" to the "poor".

Example – Factor $12x^2 + 20x + 7$

If we wish to factor using a more formal, complete and reliable method then we can reverse all of the steps of expansion algebraically.

First, let's show all of the steps of expanding: $(6x-5)(2x+3)$

$$= (6x)(2x+3)+(-5)(2x+3)$$

$$= 12x^2+18x-10x-15$$

$$= 12x^2+8x-15$$

Next, let's reverse all of the steps of expanding:

Method of decomposition and grouping

1. Determine two numbers b_1 and b_2 that have the correct product and sum.
2. Break up the middle term using b_1 and b_2.
3. Common factor by grouping.
4. Common factor again.

Example – Factor $6x^2-23x+20$ by decomposition and grouping.

Homework – Please complete questions # ___ on page ___.