MPM2D - Exam Review Package - Solutions.

1.

	Expression	Degree	Type of Polynomial
a)	$3x^2 - 2x$	2	binomial
	$4a^2b^3$	5	monomial
	$8 + 2x^4y^4 + 3x^3y^3$	8	trinomial
	$4x^5y - 2x^3y^2 + x^2y^3 + y$	6	polynomial

2.

a) $(6y-2)+(2y+8)$
$= 6y - 2 + 2y + 8$
$= 8y + 6$

b) $(8+6x)-(9+x)$
$= 8 + 6x - 9 - x$
$= -1 + 5x$

c. $(3x^2+2x-6)+(2x^2+4x+7)$
$= 3x^2 + 2x - 6 + 2x^2 - 4x + 7$
$= 5x^2 - 2x + 1$

d) $(5a^2b+2ab-3b^2)-(6a^2b-3ab+b^2)$
$= 5a^2b + 2ab - 3b^2 - 6a^2b + 3ab - b^2$
$= -a^2b + 5ab - 4b^2$

e. $(3ab)(-2ab^2)(2a^3)$
$= 3(-2)(2)(a)(a)(a^3)(b)(b^2)$
$= -12a^5b^3$

f) $(-6x^2yz)(-5y^3z)$
$= (-6)(-5)(x^2)(y)(y^3)(z)(z)$
$= 30x^2y^4z^2$

g. $\frac{-21x^2y^2z}{-7xy^2z}$
$= 3x$

h) $\frac{-32p^2q^4}{8p^2q^3}$
$= -4q$

3a. $4m(m^2 - mn - n^2) - 2n(6m^2 + mn + 4n^2)$
$= 4m^3 - 4m^2n - 4mn^2 - 12m^2n - 2mn^2 - 8n^3$
$= 4m^3 - 16m^2n - 6mn^2 - 8n^3$

3b. $2(m-3)(m+8)$
$= 2(m^2+8m-3m-24)$
$= 2(m^2+5m-24)$
$= 2m^2+10m-48$

c) $3(6x-2y)(2x-3y)$
$= 3(12x^2-18xy-4xy+6y^2)$
$= 3(12x^2-22xy+6y^2)$
$= 36x^2-66xy+18y^2$

d. $(y-4)(y-3)-(y-2)(y-5)$
$= (y^2-3y-4y+12)-(y^2-5y-2y+10)$
$= (y^2-7y+12)-(y^2-7y+10)$
$= y^2-7y+12-y^2+7y-10$
$= 2$

e) $6(m-2)(m+3)-3(3m-4)$
$= 6(m^2+3m-2m-6)-9m+12$
$= 6m^2+18m-12m-36-9m+12$
$= 6m^2-3m-24$

f. $(x+4)^2$
$= (x+4)(x+4)$
$= x^2+4x+4x+16$
$= x^2+8x+16$

g. $(y-7)^2$
$= (y-7)(y-7)$
$= y^2-7y-7y+49$
$= y^2-14y+49$

h. $(x-5)(x+5)$
$= x^2+5x-5x-25$
$= x^2-25$

i. $(5m+2n)(5m-2n)$
$= 25m^2-10mn+10mn-4n^2$
$= 25m^2-4n^2$

j) $3(2b-1)^2-2(4b-5)^2$
$= 3(2b-1)(2b-1)-2(4b-5)(4b-5)$
$= 3(4b^2-2b-2b+1)-2(16b^2-20b-20b+25)$
$= 3(4b^2-4b+1)-2(16b^2-40b+25)$
$= 12b^2-12b+3-32b^2+80b-50$
$= -20b^2+68b-47$

k. $4x^2-(2-3x)^2+6(2x-1)(2x+1)$
$= 4x^2-(2-3x)(2-3x)+6(2x-1)(2x+1)$
$= 4x^2-(4-6x-6x+9x^2)+6(4x^2+2x-2x-1)$
$= 4x^2-4+6x+6x-9x^2+24x^2+12x-12x-6$
$= 19x^2+12x-10$

l. $-5(x-6)^2-8$
$= -5(x-6)(x-6)-8$
$= -5(x^2-6x-6x+36)-8$
$= -5x^2+30x+30x-180-8$
$= -5x^2+60x-188$

4.

Eq^{n}	Vertex	Direc^{n} of Opening	Eq^{n} of Axis of Symmetry	Max or Min Point	Max or Min Value
$y=(x+3)^2-2$	$(-3,-2)$	up	$x=-3$	min	$y=-2$
$y=-(x-4)^2-3$	$(4,-3)$	down	$x=4$	max	$y=-3$
$y=2(x-1)^2+1$	$(1,1)$	up	$x=1$	min	$y=1$
$y=-\frac{1}{2}(x+6)^2+5$	$(-6,5)$	down	$x=-6$	max	$y=5$

5.

Eq^{n}	Domain	Range
$y=(x+3)^2-2$	$D=\{x\in\mathbb{R}\}$	$R=\{y\geq -2, y\in\mathbb{R}\}$
$y=-(x-4)^2-3$	$D=\{x\in\mathbb{R}\}$	$R=\{y\leq -3, y\in\mathbb{R}\}$
$y=2(x-1)^2+1$	$D=\{x\in\mathbb{R}\}$	$R=\{y\geq 1, y\in\mathbb{R}\}$
$y=-\frac{1}{2}(x+6)^2+5$	$D=\{x\in\mathbb{R}\}$	$R=\{y\leq 5, y\in\mathbb{R}\}$

6. $V(3,-1)$
VS x 2
opens ↓

$y=a(x-h)^2+k$
$=-2(x-3)^2-1$

7. $V(2,1)$
opens ↓
over 1, down 1 } normal
over 2, down 4 } step pattern

$y=a(x-h)^2+k$
$=-(x-2)^2+1$

8. $50r^2-72$
$=2(25r^2-36)$
$=2(5r-6)(5r+6)$

b. $2ax+10ay-8az$
$=2a(x+5y-4z)$

c. $3x^3y^2-12x^2y^3+18x^2y+15xy^2$
$=3xy(x^2y-4xy^2+6x+5y)$

8d. $3x(y-z)-2(y-z)$
$=(y-z)(3x-2)$

e) $4x(r+6)-(r+6)$
$=(r+6)(4x-1)$

f. x^2-5x+6 $P=6$ $S=-5$
$=(x-2)(x-3)$ $-2,-3$

g) a^2+6a+5 $P=5$ $S=6$
$=(a+5)(a+1)$ $5,1$

h. $x^2-5x-66$ $P=-66$ $S=-5$
$=(x-11)(x+6)$ $-11,6$

i) $m^2+12m+32$ $P=32$ $S=12$
$=(m+8)(m+4)$ $8,4$

j. $4x^2-16x-48$
$=4(x^2-4x-12)$ $P=-12$ $S=-4$
$=4(x-6)(x+2)$ $-6,2$

k) $2x^2-16y-66$
$=2(x^2-8x-33)$ $P=-33$ $S=-8$
$=2(x-11)(x+3)$ $-11,3$

l. $3y^2+y-4$ $P=-12$ $S=1$
$=(3y+4)(y-1)$ $4,-3$

m) $20x^2-7x-6$ $P=-120$ $S=-7$
$=(4x+1)(5x-3)$ $5,-12$

1, 324
2, 162
3, 108
4, 81
6, 54
9, 36
(12, 27)

n. $18y^2+15y-18$ $P=-324$ $S=15$
$=(9y-6)(2y+3)$ $27,-12$
$=3(3y-2)(2y+3)$

o) $8m^2+6m-20$
$=2(4m^2+3m-10)$ $P=-40$ $S=3$
$=2(4m-5)(m+2)$ $8,-5$

p. $15x^2-13x-2$ $P=-30$ $S=-13$
$=(15x+2)(x-1)$ $-15,+2$

q) $9x^2+3x-20$ $P=-180$ $S=3$
$=(3x+5)(3x-4)$ $-12,15$

r. x^2-25
$=(x-5)(x+5)$

s) $49-64m^2$
$=(7-8m)(7+8m)$

t) $81x^2-121$
$=(9x-11)(9x+11)$

u. $16a^2+40a+25$
$=(4a+5)(4a-5)$

v) $4x^2-36$
$=4(x^2-9)$
$=4(x-3)(x+3)$

w) $36x^2-81y^2$
$=(6x-9y)(6x+9y)$
$=3(2x-3y)3(2x+3y)$
$=9(2x-3y)(2x+3y)$

9a. $y = 3x^2 - 18x + 1$

$= 3(x^2 - 6x) + 1$

$= 3(x^2 - 3x - 3x + 9 - 9) + 1$

$= 3(x^2 - 3x - 3x + 9) - 27 + 1$

$= 3(x-3)^2 - 26$

min of $y = -26$ when $x = 3$

b) $y = -4x^2 - 32x - 11$

$= -4(x^2 + 8x) - 11$

$= -4(x^2 + 4x + 4x + 16 - 16) - 11$

$= -4(x^2 + 4x + 4x + 16) + 64 - 11$

$= -4(x+4)^2 + 53$

max of $y = 53$ when $x = -4$

c. $y = -7x^2 + 84x + 19$

$= -7(x^2 - 12x) + 19$

$= -7(x^2 - 6x - 6x + 36 - 36) + 19$

$= -7(x^2 - 6x - 6x + 36) + 252 + 19$

$= -7(x-6)^2 + 271$

max of $y = 271$ when $x = 6$

d) $y = 4x^2 - 40x + 7$

$= 4(x^2 - 10x) + 7$

$= 4(x^2 - 5x - 5x + 25 - 25) + 7$

$= 4(x^2 - 5x - 5x + 25) - 100 + 7$

$= 4(x-5)^2 - 93$

min of $y = -93$ when $x = 5$

10. $3(x-7)^2 - 15 = 0$

$\frac{3(x-7)^2}{3} = \frac{15}{3}$

$(x-7)^2 = 5$

$\sqrt{(x-7)^2} = \sqrt{5}$

$x - 7 \doteq \pm 2.24$

$x - 7 \doteq 2.24$ $\quad x - 7 \doteq -2.24$

$x_1 \doteq 9.24$ $\quad x_2 \doteq 4.76$

b) $2x^2 + 4x - 30 = 0$ $\quad P = -60 \div 4$ $\quad 10, -6$

$(2x + 10)(x - 3) = 0$

$2(x+5)(x-3) = 0$

$x_1 = 5 \quad x_2 = 3$

c) $3x^2 - 2x - 11 = x^2 - 5x$ $\quad P = -22 \div 3$ $\quad x$

$3x^2 - x^2 - 2x + 5x - 11 = 0$

$2x^2 + 3x - 11 = 0$

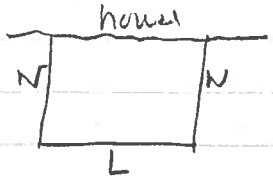
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-11)}}{2(2)}$

$= \frac{-3 \pm \sqrt{9 + 88}}{4}$

$x_1 = \frac{-3 + \sqrt{97}}{4}$ $\quad x_2 = \frac{-3 - \sqrt{97}}{4}$

$x_1 \doteq 1.71$ $\quad x_2 \doteq -3.21$

11. 

Let W & L be the dimensions.

$18 = 2W + L$

$18 - 2W = L$

$A = LW$

$= (18-2W)(W)$

$= 18W - 2W^2$

$= -2W^2 + 18W$

$W = \frac{-b}{2a}$

$= \frac{-18}{2(-2)}$

$= 4.5\ m$

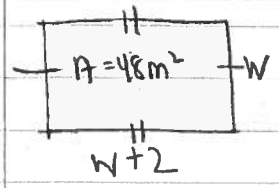
$L = 18 - 2W$

$= 18 - 2(4.5)$

$= 18 - 9$

$= 9\ m$

∴ The garden should be 9m by 4.5m.

12. 

Let W and W+2 be the dimensions

$A = LW$

$48 = (W+2)(W)$

$48 = W^2 + 2W$

$0 = W^2 + 2W - 48$

$0 = (W+8)(W-6)$

$W = -8 \quad W = 6$

↓ IMP

$L = W + 2$

$= 6 + 2$

$= 8$

∴ The dimensions are 6m by 8m.

13. Let x, $x+1$, and $x+2$ be the integers.

$(x)^2 + (x+1)^2 + (x+2)^2 = 77$

$x^2 + (x+1)(x+1) + (x+2)(x+2) = 77$

$x^2 + x^2 + x + x + 1 + x^2 + 2x + 2x + 4 = 0$

$3x^2 + 6x + 5 - 77 = 0$

$3x^2 + 6x - 72 = 0$

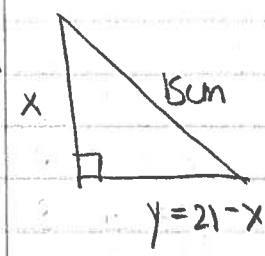
$3(x^2 + 2x - 24) = 0$

$3(x+6)(x-4) = 0$

$x_1 = -6 \quad x_2 = 4$

$x = -6$	$x = 4$
$x+1 = -5$	$x+1 = 5$
$x+2 = -4$	$x+2 = 6$

∴ The #s are 4, 5, 6 and −4, 5, −6.

14. 

$x+y=21$
$y=21-x$

Let x and 21-x be the side lengths.

$(21-x)(21-x)$
$=441-21x-21x+x^2$
$=x^2-42x+441$

$a^2+b^2=c^2$
$x^2+(21-x)^2=15^2$
$x^2+x^2-42x+441=225$
$2x^2-42x+216=0$
$2(x^2-21x+108)=0$
$2(x-12)(x-9)=0$

$P=108 \quad S=-21$
$-12,-9$

$x=12 \quad x=9$

$x=12 \quad x=9$

$y=21-x \quad y=21-x$
$=21-12 \quad =21-9$
$=9 \quad =12$

∴ The side lengths are 9cm and 12cm.

15. Let x, $x+2$, and $x+4$ be the #s.

$(x)^2+(x+2)^2+(x+4)^2=875$
$x^2+(x+2)(x+2)+(x+4)(x+4)=875$
$x^2+x^2+2x+2x+4+x^2+4x+4x+16=0$
$3x^2+12x+20-875=0$
$3x^2+12x-855=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-12\pm\sqrt{(12)^2-4(3)(-855)}}{2(3)}$$
$$=\frac{-12\pm\sqrt{144+10260}}{6}$$

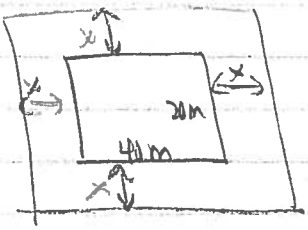
$$x_1=\frac{-12+\sqrt{10404}}{6} \qquad x_2=\frac{-12-\sqrt{10404}}{6}$$

$x_1=15$
$x+2=17$
$x+4=19$

$x_2=-19$
$x+2=-17$
$x+4=-15$

∴ The #s are 15, 17, 19 and -15, -17, & -19.

16.



Let x be the uniform width added onto each side

$A_i = LW$
$= 40(20)$
$= 800m^2$

$A_T = 2A_i$
$= 2(800)$
$= 1600m^2$

$A_T = LW$
$1600 = (40+2x)(20+2x)$
$1600 = 800 + 80x + 40x + 4x^2$
$0 = 4x^2 + 120x - 800$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$= \frac{-120 \pm \sqrt{(120)^2 - 4(4)(-800)}}{2(4)}$
$= \frac{-120 \pm \sqrt{14400 + 12800}}{8}$

$x_1 = \frac{-120 + \sqrt{27200}}{8}$ $x_2 = \frac{-120 - \sqrt{27200}}{8}$

$x_1 = 5.62$ $x_2 = -35.62$ ↓ imp.

∴ The uniform width is 5.62m.

17. $P = -2x^2 + 20x - 42$

a) $x = \frac{-b}{2a}$
$= \frac{-20}{2(-2)}$
$= \frac{-20}{-4}$
$= 5$

∴ 500 000 pairs of shoes.

b) $P = -2x^2 + 20x - 42$
$= -2(5)^2 + 20(5) - 42$
$= -2(25) + 100 - 42$
$= -50 + 100 - 42$
$= 8$

∴ $800 000 will be the profit

18 $h = -4.9(t-2)^2 + 18$

a) max height is 18m.

b) max height reached at 2 seconds.

c) $t=0$ $h = -4.9(t-2)^2 + 25$
$= -4.9(0-2)^2 + 25$
$= -4.9(-2)^2 + 25$
$= -4.9(4) + 25$
$= -19.6 + 25$
$= 5.4\text{ m}$

d. $h=0$ $h = -4.9(t-2)^2 + 25$
$0 = -4.9(t-2)^2 + 25$
$\frac{-25}{-4.9} = \frac{-4.9(t-2)^2}{-4.9}$
$5.10 = (t-2)^2$
$\sqrt{5.10} = \sqrt{(t-2)^2}$
$\pm 2.26 = t-2$

$t-2 = 2.26$ $t = 4.26$
$t-2 = -2.26$ $t = -0.26$ ↓ IMP

∴ It lands a/m 4.26 seconds

19. $y = -0.00875x^2 + 0.775x + 13$

a) $x = \frac{-b}{2a}$
$= \frac{-0.775}{2(-0.00875)}$
$= 44.29$ miles per hr

b) $y = -0.00875(44.29)^2 + 0.775(44.29) + 13$
$= -17.16403588 + 34.32475 + 13$
$= 30.16071412$
$y = 30.16$ miles per gallon.

20. $(x-2)(x+7)=0$
$x_1=2 \quad x_2=-7$

b) $6x^2-7x-3=0$
$(2x-3)(3x+1)=0$
$x_1=3/2 \quad x_2=-1/3$

$P=-18 \quad S=-7$
$-9, 2$

$2x-3=0 \quad 3x+1=0$
$2x=3 \quad 3x=-1$
$x=3/2 \quad x=-\frac{1}{3}$

c. $7x^2-35x=0$
$7x(x-5)=0$
$x_1=0 \quad x_2=5$

d) $(2x+5)(2x+5)=0$
$x=-5/2$

e. $x^2-6x+8=0$
$(x-4)(x-2)=0$
$x_1=4 \quad x_2=2$

f) $6x^2=x+35$
$6x^2-x-35=0$
$(2x-5)(3x+7)=0$
$x_1=5/2 \quad x_2=-7/3$

$P=-210 \quad S=-1$
$14, -15$

g. $\frac{5x^2}{4}-5=\frac{15x}{4}$ $\times 4$ $\times 4$

$5x^2-20=15x$
$5x^2-15x-20=0$
$5(x^2-3x-4)=0$
$5(x-4)(x+1)=0$
$x_1=4 \quad x_2=-1$

h) $x^2-8x+12=0$
$(x-6)(x-2)=0$
$x_1=6 \quad x_2=2$

i) $3x^2-6x-8=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-(-6)\pm\sqrt{(-6)^2-4(3)(-8)}}{2(3)}$

$=\frac{6\pm\sqrt{36+96}}{6}$

$x_1=\frac{6+\sqrt{132}}{6} \quad x_2=\frac{6-\sqrt{132}}{6}$

$x_1\doteq 2.91 \quad x_2=-0.91$

$P=-24 \quad S=-6$
x

j. $4x^2+32x-3=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-32\pm\sqrt{(32)^2-4(4)(-3)}}{2(4)}$

$=\frac{-32\pm\sqrt{1024+48}}{8}$

$x_1=\frac{-32+\sqrt{1072}}{8} \quad x_2=\frac{-32-\sqrt{1072}}{8}$

$x_1\doteq 0.09 \quad x_2\doteq -8.09$

$P=-12 \quad S=32$

21a) $y = (x-3)(x-5)$

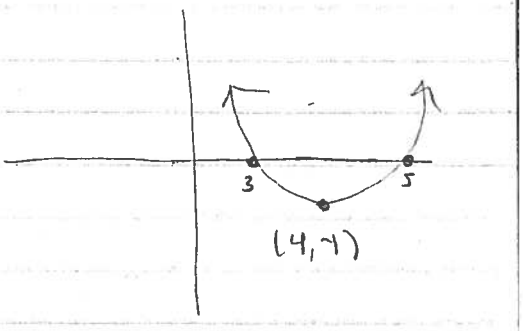
$x_1 = 3 \quad x_2 = 5$

$x = \frac{3+5}{2}$ $\quad y = (x-3)(x-5)$ $\quad V(4,-1)$

$= 8/2$ $\quad = (4-3)(4-5)$

$= 4$ $\quad = 1(-1)$

$= -1$



b) $y = x^2 - 7x + 12$

$= (x-4)(x-3)$

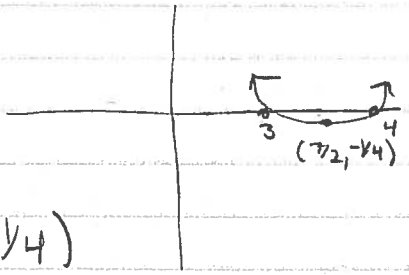
$x_1 = 4 \quad x_2 = 3$

$x = \frac{4+3}{2}$ $\quad y = (7/2 - 4)(7/2 - 3)$ $\quad V(7/2, -1/4)$

$= 7/2$ $\quad = (7/2 - 8/2)(7/2 - 6/2)$ $\quad 3.5, -0.25$

$= (-1/2)(1/2)$

$= -1/4$



c. $y = -(x-1)(x-9)$

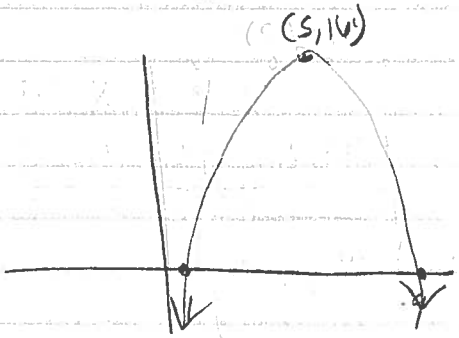
$x_1 = 1 \quad x_2 = 9$

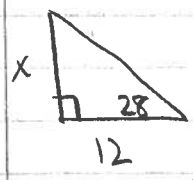
$x = \frac{1+9}{2}$ $\quad y = -(5-1)(5-9)$ $\quad V(5,16)$

$= 10/2$ $\quad = -(4)(-4)$

$= 5$ $\quad = -(-16)$

$= 16$

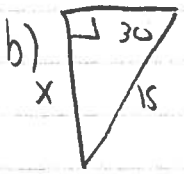


22. 

$\sin 28 = x/12$

$12 \sin 28 = x$

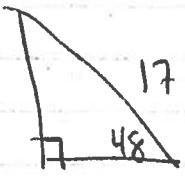
$x \doteq 5.63\text{m}$

b) 

$\sin 30 = \frac{x}{15}$

$15 \sin 30 = x$

$x = 7.5\text{m}$

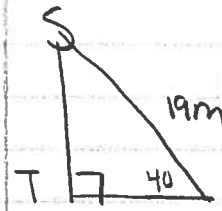
c) 

$\cos 48 = \frac{x}{17}$

$17 \cos 48 = x$

$x \doteq 11.38\text{m}$

23



$S = ?$
$T = 90°$
$U = 40°$

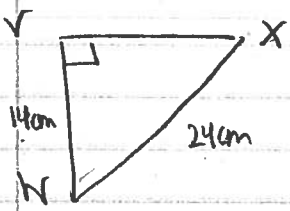
$S = ?$
$t = 19m$
$u = ?$

$S = 180 - 90 - 40$
$= 50°$
(ASTT)

$\cos 40 = \frac{s}{19}$
$s = 19 \cos 40$
$s \doteq 14.55m$

$\sin 40 = \frac{u}{19}$
$u = 19 \sin 40$
$u \doteq 12.21m$

b.



$V = 90°$
$W =$
$X =$

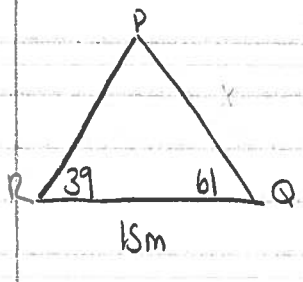
$v = 24cm$
$w \doteq$
$x = 14cm$

$x^2 + w^2 = v^2$
$14^2 + w^2 = 24^2$
$w^2 = 576 - 196$
$= 380$
$w = \sqrt{380}$
$w \doteq 19.49cm$

$\sin X = \frac{14}{24}$
$X = \sin^{-1}\left(\frac{14}{24}\right)$
$X \doteq 36°$

$\cos W = \frac{14}{24}$
$W = \cos^{-1}\left(\frac{14}{24}\right)$
$W \doteq 54°$

24



$P = ?$
$Q = 61°$
$R = 39°$

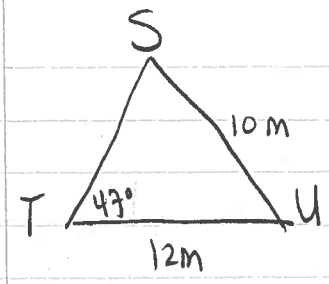
$p = 15m$
$q = ?$
$r = ?$

$P = 180 - 61 - 39$
$= 80°$
(ASTT)

$\frac{r}{\sin R} = \frac{p}{\sin P}$
$\frac{r}{\sin 39} = \frac{15}{\sin 80}$
$r = \frac{15 \sin 39}{\sin 80}$
$r \doteq 9.59m$

$\frac{q}{\sin Q} = \frac{p}{\sin P}$
$\frac{q}{\sin 61} = \frac{15}{\sin 80}$
$q = \frac{15 \sin 61}{\sin 80}$
$q \doteq 13.32m$

b.



S =
T = 47°
U =

s = 12m
t = 10m
u =

$\frac{\sin S}{s} = \frac{\sin T}{t}$

$\frac{\sin S}{12} = \frac{\sin 47}{10}$

$S = \sin^{-1}\left(\frac{12 \sin 47}{10}\right)$

$S \doteq 61°$

$U = 180 - 47 - 61$
$= 72°$

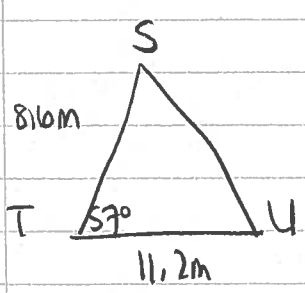
$\frac{u}{\sin U} = \frac{t}{\sin T}$

$\frac{u}{\sin 72} = \frac{10}{\sin 47}$

$u = \frac{10 \sin 72}{\sin 47}$

$u \doteq 13.00\ m$

c.



S = ?
T = 57°
U = ?

s = 11.2m
t = ?
u = 8.16m

$t^2 = s^2 + u^2 - 2su\cos T$

$= (11.2)^2 + (8.16)^2 - 2(11.2)(8.16)\cos 57$

$t = \sqrt{199.4 - 192.64\cos 57}$

$t \doteq 9.72\ m$

$\frac{\sin S}{s} = \frac{\sin T}{t}$

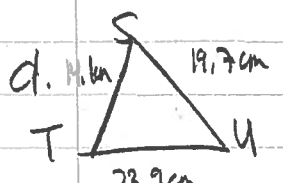
$\frac{\sin S}{11.2} = \frac{\sin 57}{9.72}$

$S = \sin^{-1}\left(\frac{11.2 \sin 57}{9.72}\right)$

$S \doteq 75°$

$U = 180 - 57 - 75$
$= 48°$
(AST)

d.



S = ?
T = ?
U = ?

s = 23.9cm
t = 19.7cm
u = 14.1cm

$s^2 = t^2 + u^2 - 2tu\cos S$

$23.9^2 = (19.7)^2 + (14.1)^2 - 2(19.7)(14.1)\cos S$

$\frac{-15.69}{-555.54} = \frac{-555.54\cos S}{-555.54}$

$S = \cos^{-1}\left(\frac{-15.69}{-555.54}\right) \Rightarrow S \doteq 88°$

$\frac{\sin T}{t} = \frac{\sin S}{s}$

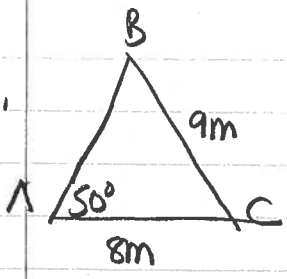
$\frac{\sin T}{19.7} = \frac{\sin 88}{23.9}$

$T = \sin^{-1}\left(\frac{19.7 \sin 88}{23.9}\right)$

$T \doteq 55°$

$U = 180 - 88 - 55$
$= 37°$
(AST)

25.



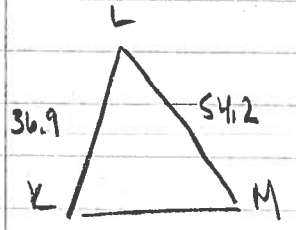
$\frac{\sin B}{b} = \frac{\sin A}{a}$

$\frac{\sin B}{8} = \frac{\sin 50}{9}$

$B = \sin^{-1}\left(\frac{8\sin 50}{9}\right)$

$B \doteq 43°$

26.



$k^2 = l^2 + m^2 - 2lm\cos K$

$(54.2)^2 = (45.7)^2 + (36.9)^2 - 2(45.7)(36.9)\cos K$

$2937.64 = 3450.1 - 3372.66\cos K$

$\frac{-512.46}{-3372.66} = \frac{-3372.66\cos K}{-3372.66}$

$K = \cos^{-1}\left(\frac{-512.46}{-3372.66}\right)$

$K \doteq 81°$

27a. $3x + y = 17$ ①

$2x - y = -2$ ②

① + ② $\frac{5x}{5} = \frac{15}{5}$

$x = 3$

Sub $x = 3$ into ①

$3(3) + y = 17$

$9 + y = 17$

$y = 17 - 9$

$y = 8$

∴ POI is $(3, 8)$

b. $3x - 4y = 5$ ①

$5x + 3y = -11$ ②

① × 3: $9x - 12y = 15$ ③

② × 4: $20x + 12y = -44$ ④

③ + ④ $\frac{29x}{29} = \frac{-29}{29}$

$x = -1$

Sub $x = -1$ into ①

$3x - 4y = 5$

$3(-1) - 4y = 5$

$-3 - 4y = 5$

$\frac{-4y}{-4} = \frac{8}{-4}$

$y = -2$

∴ POI is $(-1, -2)$

27c. $\frac{x}{6} + \frac{y}{4} = 6$ ①

$\frac{5x}{6} - \frac{y}{3} = 11$ ②

① × 12: $2x + 3y = 72$ ③

② × 6: $5x - 2y = 66$ ④

③ × 2: $4x + 6y = 144$ ⑤

④ × 3: $15x - 6y = 198$ ⑥

⑤ + ⑥: $\frac{19x}{19} = \frac{342}{19}$

$x = 18$

Sub $x = 18$ into ②

$2x + 3y = 72$

$2(18) + 3y = 72$

$36 + 3y = 72$

$3y = 72 - 36$

$\frac{3y}{3} = \frac{36}{3}$

$y = 12$.

∴ POI is (18, 12)

28. Let x be the # of 2kg bags of sugar.
" y " " " " 4kg " " "

$x + y = 1100$ ①

$2x + 4y = 2900$ ②

b. Let x be the # of cars washed
" y " " " " vans "

$x + y = 86$ ①

$5x + 6y = 475$ ②

c.

x	+	y	=	5
0.30		0.35		0.33
0.30x		0.35y		1.65

Let x be the amnt of 30% sol'n used
" y " " " " 35% " "

$x + y = 5$ ①

$0.30x + 0.35y = 1.65$ ②

d. let x be the # of chkn dinners prepared
" y " " " " " beef " " "

$$x + y = 300$$
$$16x + 18y = 5256.$$

e.

100%	+	20%	=	25%
1		0.2		0.25
x		18		y
x		3.6		0.25y

$$x + 18 = y \quad (1)$$
$$x + 3.6 = 0.25y \quad (2)$$

29. $S(3,8)$ $y = \frac{2}{7}x + \frac{18}{7}$

1) $m = 2/7$ 2) $m_\perp = -7/2$ 3) $y = mx + b$.

$8 = -7/2(3/1) + b$

$8 = -21/2 + b$

$b = 16/2 + 21/2$

$b = 37/2$

$y = -\frac{7}{2}x + \frac{37}{2}$

4) POI: $y = \frac{2}{7}x + \frac{18}{7}$ (1)

$y = -\frac{7}{2}x + \frac{37}{2}$ (2)

(1) x 7: $7y = 2x + 18$ (3)

(2) x 2: $2y = -7x + 37$ (4)

(3) x 7: $49y = 14x + 126$ (5)

(4) x 2: $4y = -14x + 74$ (6)

(5) + (6): $\frac{53y}{53} = \frac{200}{53}$

$y = 200/53$

Sub $y = 200/53$ into (3)

$7(200/53) = 2x + 18$

$1400/53 - 18/1 = 2x$

$1400/53 - 954/53 = 2x$

$446/53 = 2x$

$446/53 \cdot 1/2 = 2x \cdot 1/2$

$446/106 = x$

$x = 223/53$

$S(3,8)$

$P(223/53, 200/53)$

5) $L^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$= [223/53 - 3/1]^2 + [200/53 - 8/1]^2$

$= [223/53 - 159/53]^2 + [200/53 - 424/53]^2$

$= (64/53)^2 + (-224/53)^2$

$= \frac{4096}{2809} + \frac{50176}{2809}$

$= \frac{54272}{2809}$ ÷53

÷53

$L = \frac{1024}{53}$

$L = \sqrt{\frac{1024}{53}}$

$L \doteq 4.40$ units

30. (2,9) $y = x - 1$ (2,9) (6,5)

1) $m = 1 \Rightarrow m_\perp = -1$

3) $y = mx + b$
$9 = -1(2) + b$
$9 = -2 + b$
$11 = b$
$y = -x + 11$

4) $y = x - 1$ ①
$y = -x + 11$ ②
① + ② $2y = 10$
$y = 5$
Sub $y = 5$ into ①
$5 = x - 1$
$x = 6$.
POI is (6,5)

5) $L^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$= [6-2]^2 + [5-9]^2$
$= (4)^2 + (-4)^2$
$= 16 + 16$
$= 32$
$L = \sqrt{32}$ units

31. $L_{AB}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$= [3 - (-1)]^2 + [7 - (-5)]^2$
$= (4)^2 + (12)^2$
$= 16 + 144$
$L_{AB} = \sqrt{160}$ units

b) $L_{CD}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$= [6 - 0]^2 + [10 - 5]^2$
$= (6)^2 + (5)^2$
$= 36 + 25$
$L = \sqrt{61}$ units

32. Centre (0,0) Point (3,4)
$x^2 + y^2 = r^2$
$(3)^2 + (4)^2 = r^2$
$9 + 16 = r^2$
$25 = r^2$
$r = 5$

$x^2 + y^2 = 25$

33. $M_{AB} = M\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$
$= M\left(\frac{-6+4}{2}, \frac{2+8}{2}\right)$
$= M\left(\frac{-2}{2}, \frac{10}{2}\right)$
$= M(-1, 5)$

b) $M_{CD} = M\left(\frac{x_C + y_D}{2}, \frac{y_C + y_D}{2}\right)$
$= M\left(\frac{-200+350}{2}, \frac{-100+600}{2}\right)$
$= M\left(\frac{150}{2}, \frac{500}{2}\right)$
$= M(75, 250)$

34. $M_x = \frac{x_1+x_2}{2}$ $M_y = \frac{y_1+y_2}{2}$

$4 = \frac{9+x_2}{2}$ $8 = \frac{-10+y_2}{2}$

$8 = 9+x_2$ $16 = -10+y_2$

$-1 = x_2$ $26 = y_2$

$\therefore$ D $(-1, 26)$

b) $M_x = \frac{x_1+x_2}{2}$ $M_y = \frac{y_1+y_2}{2}$

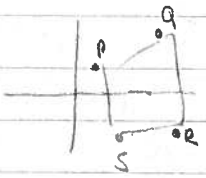
$10 = \frac{-9+x_2}{2}$ $-3 = \frac{7+y_2}{2}$

$20 = -9+x_2$ $-6 = 7+y_2$

$29 = x_2$ $-13 = y_2$

D $(29, -13)$

35. P(2,3)
Q(8,5)
R(11,-4)
S(5,-6)



$m_{PQ} = \frac{y_2-y_1}{x_2-x_1} = \frac{5-3}{8-2} = \frac{2}{6} = \frac{1}{3}$

$m_{QR} = \frac{y_2-y_1}{x_2-x_1} = \frac{-4-5}{11-8} = \frac{-9}{3} = -3$

$m_{RS} = \frac{y_2-y_1}{x_2-x_1} = \frac{-4-(-6)}{11-5} = \frac{2}{6} = \frac{1}{3}$

$m_{PS} = \frac{y_2-y_1}{x_2-x_1} = \frac{-6-3}{5-2} = \frac{-9}{3} = -3$

$L_{PQ}^2 = (x_2-x_1)^2 + (y_2-y_1)^2$
$= [8-2]^2 + [5-3]^2$
$= (6)^2 + (2)^2$
$= 36+4$
$L_{PQ} = \sqrt{40}$ units

$L_{QR}^2 = (x_2-x_1)^2 + (y_2-y_1)^2$
$= [11-8]^2 + [-4-5]^2$
$= (3)^2 + (-9)^2$
$= 9+81$
$L_{QR} = \sqrt{90}$ units

$L_{RS}^2 = (x_2-x_1)^2 + (y_2-y_1)^2$
$= [11-5]^2 + [-4-(-6)]^2$
$= (6)^2 + (2)^2$
$= 36+4$
$L_{RS} = \sqrt{40}$ units

$L_{PS}^2 = (x_2-x_1)^2 + (y_2-y_1)^2$
$= [5-2]^2 + [-6-3]^2$
$= [3]^2 + [-9]^2$
$= 9+81$
$L_{PS} = \sqrt{90}$ units

Opposite sides are parallel & have equal length.
All corners are 90°
Not all 4 sides are equal length

$\therefore$ Rectangle.

36. T(10,-3)
U(-10,9)
V(-12,-5)

$m_{TU} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-3-9}{10-(-10)}$

$= \frac{-12}{20}$

$= -\frac{3}{5}$

$m_\perp = 5/3$

$y = mx + b$

$y = -\frac{3}{5}x + 3$

(got b from graph)

$y = mx + b$

$-5 = \frac{5}{3}(-12) + b$

$-5 = -60/3 + b$

$-5 = -20 + b$

$b = 15$

$y = \frac{5}{3}x + 15.$

POI: $y = -3/5x + 3$ ①

$y = 5/3x + 15$ ②

①×5: $5y = -3x + 15$ ③

②×3: $3y = 5x + 45$ ④

③×5: $25y = -15x + 75$ ⑤

④×3 $9y = 15x + 135$ ⑥

⑤+⑥ $34y = 210$

$y = \frac{210}{34}$

$y = 105/17$

Sub $y = 105/17$ into ③

$5\left(\frac{105}{17}\right) = -3x + 15$

$525/17 - 255/17 = -3x$

$270/17 = -3x$

$270/17 \cdot -1/3 = -3x(-1/3)$

$-270/51 = x$

$x = -\frac{90}{17}$

$\therefore$ POI $\left(-\frac{90}{17}, \frac{105}{17}\right)$

V(-12,-5)

POI $\left(\frac{-90}{17}, \frac{105}{17}\right)$

base: $L_{TU}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$= [10-(-10)]^2 + [-3-9]^2$

$= (20)^2 + (-12)^2$

$= \sqrt{544}$ units.

height: $L_{VP}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$= \left[-\frac{90}{17} - (-12)\right]^2 + \left[\frac{105}{17} - (-5)\right]^2$

$= \left[-\frac{90}{17} + \frac{204}{17}\right]^2 + \left[\frac{105}{17} + \frac{85}{17}\right]^2$

$= \left[114/17\right]^2 + \left[190/17\right]^2$

$= 12996/289 + 36100/289$

$L_{VP} = \sqrt{\frac{49096}{289}}$ units

$A = \frac{bh}{2}$

$= \frac{\sqrt{544} \cdot \sqrt{\frac{49096}{289}}}{2}$

$= \frac{304}{2}$

$= 152$ units^2

37. A(-11, 2)
B(-7, 8)
C(3, 12).

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{8-2}{-7-(-11)}$
$= \frac{6}{4}$
$= 3/2$
$\boxed{m_\perp = -2/3}$

$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{12-8}{3-(-7)}$
$= \frac{4}{10}$
$= 2/5$
$\boxed{m_\perp = -5/2}$

$M_{AB} = M\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$
$= M\left(\frac{-11+(-7)}{2}, \frac{2+8}{2}\right)$
$= M\left(\frac{-18}{2}, \frac{10}{2}\right)$
$= M(-9, 5)$

$M_{BC} = M\left(\frac{x_B + x_C}{2}, \frac{y_B + y_C}{2}\right)$
$= M\left(\frac{-7+3}{2}, \frac{8+12}{2}\right)$
$= M\left(\frac{-4}{2}, \frac{20}{2}\right)$
$= M(-2, 10)$

$y = mx + b$
$5 = -2/3(-9/1) + b$
$5 = 18/3 + b$
$5 = 6 + b$
$b = -1$
$y = -\frac{2}{3}x - 1$

$y = mx + b$
$10 = -5/2(-2) + b$
$10 = +10/2 + b$
$10 = 5 + b$
$b = 5$
$y = -5/2x + 5$

POI: $y = -2/3x - 1$ (1)
$y = -5/2x + 5$ (2)
(1) × 3: $3y = -2x - 3$ (3)
(2) × 2: $2y = -5x + 10$ (4)
(3) × -2: $-6y = 4x + 6$ (5)
(4) × 3: $6y = -15x + 30$ (6)
(5) + (6): $0 = -11x + 36$
$\frac{11x}{11} = \frac{36}{11}$
$x = \frac{36}{11}$

Sub $x = 36/11$ into (3)
$3y = -2(36/11) - 3$
$3y = -72/11 - 3$
$3y = -72/11 - 33/11$
$3y = -105/11$
$3y \cdot 1/3 = -105/11 \cdot 1/3$
$y = -105/33$

∴ Centre of circle is
$\left(\frac{36}{11}, \frac{-105}{33}\right)$

4. Complete the table below and then graph the functions on the grid provided.

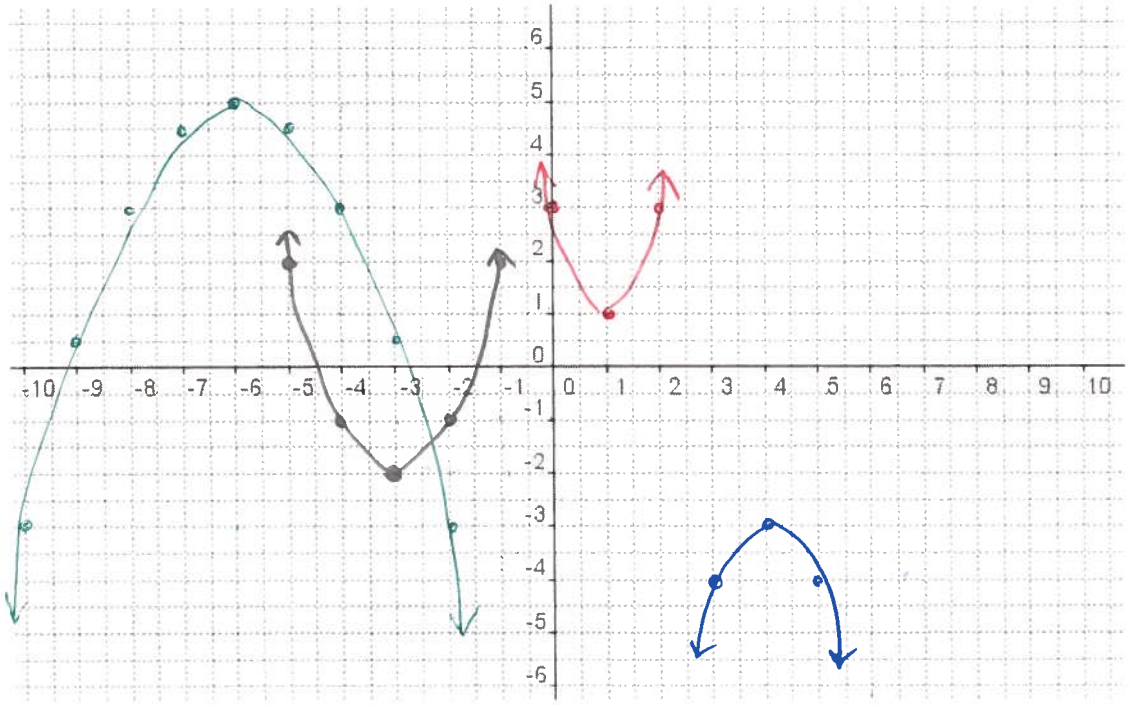
Equation	Vertex	Direction of Opening	Equation of the Axis of Symmetry	Maximum or Minimum Point	Maximum or Minimum Value
$y=(x+3)^2-2$	$(-3,-2)$	up	$x=-3$	min	$y=-2$
$y=-(x-4)^2-3$	$(4,-3)$	down	$x=4$	max	$y=-3$
$y=2(x-1)^2+1$	$(1,1)$	up	$x=1$	min	$y=1$
$y=-\frac{1}{2}(x+6)^2+5$	$(-6,5)$	down	$x=-6$	max	$y=5$

a) $y=(x+3)^2-2$

b) $y=-(x-4)^2-3$

c) $y=2(x-1)^2+1$

d) $y=-\frac{1}{2}(x+6)^2+5$



5. Complete the table below.

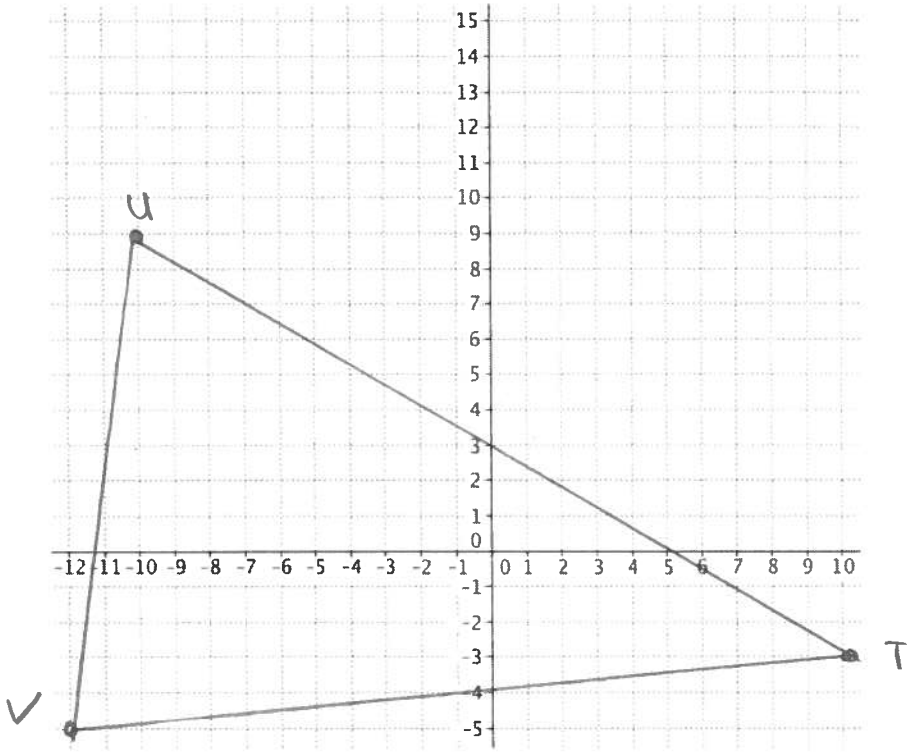
Equation	Domain	Range
$y=(x+3)^2-2$	$D=\{x\in\mathbb{R}\}$	$R=\{y\geq -2, y\in\mathbb{R}\}$
$y=-(x-4)^2-3$	$D=\{x\in\mathbb{R}\}$	$R=\{y\leq -3, y\in\mathbb{R}\}$
$y=2(x-1)^2+1$	$D=\{x\in\mathbb{R}\}$	$R=\{y\geq 1, y\in\mathbb{R}\}$
$y=-\frac{1}{2}(x+6)^2+5$	$D=\{x\in\mathbb{R}\}$	$R=\{y\leq 5, y\in\mathbb{R}\}$

6. Write an equation for a quadratic relation that has a vertex (3, -1) and has been vertically stretched by a factor of 2 and opens down.

34. Find the second endpoint of a line segment given the midpoint, M, and one endpoint, C.
    a. C (9, -10) and M (4, 8)
    b. C (-9, 7) and M (10, -3)

35. A quadrilateral PQRS has vetices P (2, 3), Q (8, 5), R (11, -4), and S (5, -6). Determine what type of quadrilateral PQRS is.

36. Determine the area of the triangle with vertices T (10, –3), U (–10, 9) and V (–12, –5).



37. Determine the centre of the circle that passes through A (–11, 2), B (–7, 8) and C (3, 12).