MPM2D - Unit 7 - Analytic Geometry

1. $A(4,3)$ $B(-2,7)$

a) $M_{AB} = M\left(\frac{x_A+x_B}{2}, \frac{y_A+y_B}{2}\right)$

$= M\left(\frac{4+(-2)}{2}, \frac{3+7}{2}\right)$

$= M(1,5)$

b) $L_{AB}^2 = (x_2-x_1)^2+(y_2-y_1)^2$

$= [4-(-2)]^2 + [3-7]^2$

$= 6^2 + (-4)^2$

$= 36+16$

$L_{AB} = \sqrt{52}$ units

$m_{AB} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{3-7}{4-(-2)}$

$= \frac{-4}{6}$

$= -2/3$

d) $y = mx+b$

$3 = -\frac{2}{3}(4)+b$

$3 = -8/3+b$

$9/3+8/3 = b$

$b = 17/3$

$y = -\frac{2}{3}x+\frac{17}{3}$

$y = mx+b$

$7 = -\frac{2}{3}(-2)+b$

$7 = 4/3+b$

$21/3 - 4/3 = b$

$b = 17/3$

$y = -\frac{2}{3}x+\frac{17}{3}$

2. $A(-1,-2)$ $B(-7,10)$

$M_{AB} = M\left(\frac{x_A+x_B}{2}, \frac{y_A+y_B}{2}\right)$

$= M\left(\frac{-1+(-7)}{2}, \frac{-2+10}{2}\right)$

$= M(-4,4)$

b) $L_{AB}^2 = (x_2-x_1)^2+(y_2-y_1)^2$

$= [-1-(-7)]^2 + [-2-10]^2$

$= (6)^2 + (-12)^2$

$= 36+144$

$L_{AB} = \sqrt{180}$ units

c) $m = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-2-10}{-1-(-7)}$

$= \frac{-12}{6}$

$= -2$

d) $y = mx+b$

$-2 = -2(-1)+b$

$-2 = 2+b$

$b = -4$

$y = -2x-4$

$y = mx+b$

$10 = -2(-7)+b$

$10 = 14+b$

$b = -4$

$y = -2x-4$

3. $A(2,1)$ $B(3,5)$

$L_{AB}^2 = (x_2-x_1)^2 + (y_2-y_1)^2$
$= [3-2]^2 + [5-1]^2$
$= 1^2 + 4^2$
$L_{AB} = \sqrt{17}$ units

b. $C(3,5)$ $D(-6,7)$

$L_{CD}^2 = (x_2-x_1)^2 + (y_2-y_1)^2$
$= [3-(-6)]^2 + [-5-7]^2$
$= (9)^2 + (-12)^2$
$L_{CD} = \sqrt{225}$
$L_{CD} = 15$ units

4. $A(5,7)$ $B(3,9)$

$M_{AB} = M\left(\frac{x_A+x_B}{2}, \frac{y_A+y_B}{2}\right)$
$= M\left(\frac{5+3}{2}, \frac{7+9}{2}\right)$
$= M(4,8)$

b) $C(1/2, 5/2)$ $D(3/2, -7/2)$

$M_{CD} = M\left(\frac{x_C+x_D}{2}, \frac{y_C+y_D}{2}\right)$
$= M\left(\frac{1/2+3/2}{2}, \frac{5/2-7/2}{2}\right)$
$= M\left(\frac{4/2}{2}, \frac{-2/2}{2}\right)$
$= M\left(1, -\frac{1}{2}\right)$

5. $C(5,-7)$ $M(3.5, 1.5)$

$M_x = \frac{x_1+x_2}{2}$ $M_y = \frac{y_1+y_2}{2}$

$3.5 = \frac{5+x_2}{2}$ $1.5 = \frac{-7+y_2}{2}$

$7 = 5+x_2$ $3 = -7+y_2$

$2 = x_2$ $10 = y_2$

$\therefore D$ is $(2,10)$

6) $J(3,5)$ $M(7,10)$

$M_x = \frac{x_1+x_2}{2}$ $M_y = \frac{y_1+y_2}{2}$

$7 = \frac{3+x_2}{2}$ $10 = \frac{5+y_2}{2}$

$14 = 3+x_2$ $20 = 5+y_2$

$11 = x_2$ $15 = y_2$

$\therefore K$ is $(11,15)$

7. You would substitute the x and y value into the eqn & solve. If
$x^2+y^2 > r^2$: the pt is outside the circle
$x^2+y^2 < r^2$: the pt is inside the circle
$x^2+y^2 = r^2$: the pt is on the circle

8. Pt $(3,6)$

$x^2+y^2=49$

$(3)^2+(6)^2=49$

$9+36=49$

$45=49$

$\because x^2+y^2<r^2$

$\therefore$ Pt is inside the circle

b) Pt $(10,5)$

$x^2+y^2=121$

$(10)^2+(5)^2=121$

$100+25=121$

$125=121$

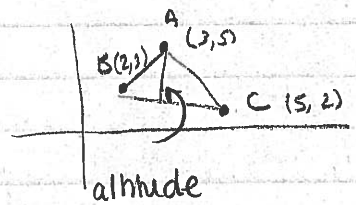
$\because x^2+y^2>r^2$

$\therefore$ Pt is outside the circle

9. $A(3,5)$

$B(2,3)$

$C(5,2)$



$m_{BC}=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{2-3}{5-2}$

$=-\frac{1}{3}$

$m_\perp=3$

$y=mx+b$

$5=3(3)+b$

$5=9+b$

$-4=b$

$\boxed{y=3x-4}$

10. Shortest Distance btwn $(6,5)$ and $7x+y+23=0$

1) m: $7x+y+23=0$

$y=-7x-23$

$m=-7$

2) $m_\perp=1/7$

3) $y=mx+b$

$5=\frac{1}{7}(6)+b$

$5=6/7+b$

$\frac{35}{7}-\frac{6}{7}=b$

$b=29/7$

$y=1/7x+29/7$

4) POI: $7x+y+23=0$ ①

$y=\frac{1}{7}x+\frac{29}{7}$ ②

②×7: $7y=x+29$ ③

from ①: $y=-7x-23$ ④

③×7: $49y=7x+203$ ⑤

④+⑤: $\frac{50y}{50}=\frac{180}{50}$

$y=18/5$ 3.6

Sub $y=18/5$ into ①

$7x+18/5+23=0$

$7x=-18/5-115/5$

$7x=-\frac{133}{5}$

$x=-\frac{133}{5}\div 7$

$x=-\frac{133}{5}\cdot\frac{1}{7}$

$=-133/35$ -3.8

POI is $\left(\frac{18}{5},-\frac{133}{35}\right)$

POI is $(3.6,-3.8)$

5) $L^2=(x_2-x_1)^2+(y_2-y_1)^2$

$=(6-3.6)^2+(5-(-3.8))^2$

$=(2.4)^2+(8.8)^2$

$=83.2$

$L=\sqrt{83.2}$ units

$\left(6-\frac{18}{5}\right)^2+\left(5+\frac{133}{35}\right)^2$

$=\left(\frac{30-18}{5}\right)^2+\left(\frac{175+133}{35}\right)^2$

$=\left(\frac{12}{5}\right)^2+\left(\frac{308}{35}\right)^2$

$=144/25+\frac{94864}{1225}$

$=101920/1225$

11 P(-1,4)
Q(8,1)
R(1,-2)

$L_{PQ}^2 = (x_2-x_1)^2+(y_2-y_1)^2$
$= [8-(-1)]^2+[1-4]^2$
$= 9^2+(-3)^2$
$= 81+9$
$L_{PQ} = \sqrt{89}$ units

$L_{QR}^2 = (x_2-x_1)^2+(y_2-y_1)^2$
$= (8-1)^2+(1-(-2))^2$
$= (7)^2+(3)^2$
$= 49+9$
$L_{QR} = \sqrt{58}$ units

$L_{PR}^2 = (x_2-x_1)^2+(y_2-y_1)^2$
$= [1-(-1)]^2+[-2-4]^2$
$= [2]^2+[-6]^2$
$= 4+36$
$L_{PR} = \sqrt{40}$ units

$\therefore L_{PQ} \neq L_{QR} \neq L_{PR}$
$\therefore$ It is a scalene triangle

b. K(2,5)
L(4,1)
M(6,-3)

$L_{KL}^2 = (x_2-x_1)^2+(y_2-y_1)^2$
$= (4-2)^2+(1-5)^2$
$= (2)^2+(-4)^2$
$= 4+16$
$L_{KL} = \sqrt{20}$ units

$L_{LM}^2 = (x_2-x_1)^2+(y_2-y_1)^2$
$= (6-4)^2+(-3-1)^2$
$= (2)^2+(-4)^2$
$= 4+16$
$L_{LM} = \sqrt{20}$ units

$L_{KM}^2 = (x_2-x_1)^2+(y_2-y_1)^2$
$= (6-2)^2+(-3-5)^2$
$= (4)^2+(-8)^2$
$= 16+64$
$L_{KM} = \sqrt{80}$ units

$\therefore L_{KL} = L_{LM} \neq L_{KM}$
$\therefore$ It is an isosceles triangle

* 2 equal side lengths

A(-8,-3)
B(22,3)
C(9,16)
D(-1,14)

12. $m_{DC} = \dfrac{y_2-y_1}{x_2-x_1}$
$= \dfrac{16-14}{9-(-1)}$
$= \dfrac{2}{10}$
$= 1/5$

$m_\perp = -5$

P(2,-1)

$L_{DC}^2 = (x_2-x_1)^2+(y_2-y_1)^2$
$= (9-(-1))^2+(16-14)^2$
$= (10)^2+(2)^2$
$L_{DC} = \sqrt{104}$ units

$L_{AB}^2 = (x_2-x_1)^2+(y_2-y_1)^2$
$= [22-(-8)]^2+[3-(-3)]^2$
$= (22+8)^2+(3+3)^2$
$= (30)^2+(6)^2$
$L_{AB} = \sqrt{936}$ units

$L_{DP}^2 = (x_2-x_1)^2+(y_2-y_1)^2$
$= (2-(-1))^2+(-1-14)^2$
$= (3)^2+(-15)^2$
$= 9+225$
$L_{DP} = \sqrt{234}$ units

$A = \left(\dfrac{a+b}{2}\right)h$
$= \dfrac{\sqrt{104}+\sqrt{936}}{2} \cdot \sqrt{234}$
$\boxed{A = 312 \text{ units}^2}$

13. $M_{PQ} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-7-2}{4-(-8)}$

$= \frac{-9}{12}$

$= \frac{-3}{4}$

$M\perp = 4/3$

POI : $T(0,-4)$

$L_{PQ}^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$= [4-(-8)]^2 + [-7-2]^2$

$= (12)^2 + (-9)^2$

$= 144 + 81$

$= 225$

$L_{PQ} = \sqrt{225}$

$L_{PQ} = 15$ units

$L_{RT}^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$= (6-0)^2 + [4-(-4)]^2$

$= (6)^2 + (8)^2$

$= 36 + 64$

$= 100$

$L_{RT} = \sqrt{100}$

$L_{RT} = 10$ units

$A = \frac{bh}{2}$

$= \frac{(15)(10)}{2}$

$\boxed{A = 75 \text{ units}^2}$

14. $M_{AB} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{2-3}{4-(-3)}$

$= \frac{-1}{7}$

Perp Bisector $M_\perp = 7$

$y = mx+b$

$1 = 7(1)+b$

$1 = 7+b$

$b = -6$

$y = 7x-6$

AB: $y = mx+b$

$3 = -\frac{1}{7}(-3)+b$

$3 = 3/7 + b$

$21/7 - 3/7 = b$

$18/7 = b$

$y = -\frac{1}{7}x + \frac{18}{7}$

$y = 7x - 6$ ①

$y = -\frac{1}{7}x + \frac{18}{7}$ ②

② × 7: $7y = -x + 18$ ③

① × -7: $-7y = -49x + 42$ ④

③ + ④: $0 = -50x + 60$

$\frac{50x}{50} = \frac{60}{50}$

$x = 6/5$

$x = 1.2$

Sub $x = 1.2$ into ①

$y = 7x-6$

$= 7(1.2)-6$

$= 8.4-6$

$= 2.4$

$\therefore$ POI is $P(1.2, 2.4)$

$C(1,1)$

$P(1.2, 2.4)$

$L_{AB}^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$= [4-(-3)]^2 + [2-3]^2$

$= (7)^2 + (-1)^2$

$= 49 + 1$

$L_{AB} = \sqrt{50}$ units

$L_{CP}^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$= [1.2-1]^2 + [2.4-1]^2$

$= (0.2)^2 + (1.4)^2$

$= 2$

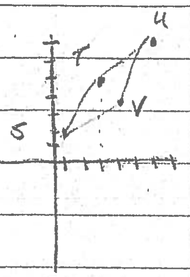
$L_{CP} = \sqrt{2}$ units

$A = \frac{bh}{2}$

$= \frac{\sqrt{50} \cdot \sqrt{2}}{2}$

$= 5 \text{ units}^2$

15. S(1,2)
T(3,5)
U(6,7)
V(4,4)



$m_{ST} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{3-1} = \frac{3}{2}$

$m_{TU} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-5}{6-3} = \frac{2}{3}$

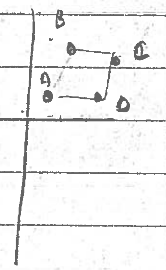
$m_{UV} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{6-4} = \frac{3}{2}$

$m_{SV} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{4-1} = \frac{2}{3}$

$\left.\begin{array}{l} m_{ST} = m_{UV} \\ m_{TU} = m_{SV} \end{array}\right\}$ 2 sets of parallel slopes.

$\therefore$ There are 2 sets of parallel slopes, it is a parallelogram.

16. A(1,2)
B(2,5)
C(5,4)
D(4,1)



$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{2-1} = 3$

$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-2}{4-1} = -\frac{1}{3}$

$m_{AB} \perp m_{AC}$ ✓

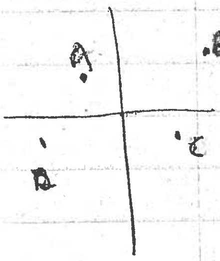
$L_{AB}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$= (2-1)^2 + (5-2)^2$
$= (1)^2 + (3)^2$
$L_{AB} = \sqrt{10}$ units

$L_{BC}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$= (5-2)^2 + (4-5)^2$
$= 3^2 + (1)^2$
$L_{BC} = \sqrt{10}$ units

$L_{CD}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$= (5-4)^2 + (4-1)^2$
$= (1)^2 + (3)^2$
$= \sqrt{10}$ units

$L_{AD}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$= (4-1)^2 + (1-2)^2$
$= 3^2 + (-1)^2$
$L_{AD} = \sqrt{10}$ units

$\therefore$ There a 2 perpendicular slopes & 4 equal side lengths, it is a square.



17. A$(-1,3)$
B$(6,4)$
C$(4,-1)$
D$(-3,-2)$

$m_{AB} = \frac{y_2-y_1}{x_2-x_1} = \frac{4-3}{6-(-1)} = \frac{1}{7}$

$m_{BC} = \frac{y_2-y_1}{x_2-x_1} = \frac{-1-4}{4-6} = \frac{-5}{-2} = 5/2$

$m_{CD} = \frac{y_2-y_1}{x_2-x_1} = \frac{-1-(-2)}{4-(-3)} = \frac{1}{7}$

$m_{AD} = \frac{y_2-y_1}{x_2-x_1} = \frac{3-(-2)}{-1-(-3)} = \frac{5}{2}$

- 2 sets of parallel slopes
  ↓
  square or rectangle or parallelogram
- Slopes are not perpendicular & ∴ its not a square or rectangle.

18. G$(-4,-9)$
H$(-2,-5)$
I$(10,-1)$

$m_{GH} = \frac{y_2-y_1}{x_2-x_1} = \frac{-5-(-9)}{-2-(-4)} = \frac{4}{2} = 2$

$m_\perp = -1/2$

$m_{HI} = \frac{y_2-y_1}{x_2-x_1} = \frac{-1-(-5)}{10-(-2)} = \frac{4}{12} = 1/3$

$m_\perp = -3$

$M_{GH} = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = M\left(\frac{-4+(-2)}{2}, \frac{-9+(-5)}{2}\right) = M(-3,-7)$

$M_{HI} = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = M\left(\frac{-2+10}{2}, \frac{-5+(-1)}{2}\right) = M(4,-3)$

$y = mx+b$
$-7 = -1/2(-3)+b$
$-7 = 3/2+b$
$-14/2 - 3/2 = b$
$b = -\frac{17}{2}$ $y = -\frac{1}{2}x - \frac{17}{2}$

$y = mx+b$
$-3 = -3(4)+b$
$-3 = -12+b$
$b = 9$
$y = -3x+9$

→

18 con't

$y = -\frac{1}{2}x - \frac{17}{2}$ ①

$y = -3x + 9$ ②

①×2 $2y = -x - 17$ ③

②×−2 $-2y = 6x - 18$ ④

③+④ $0 = 5x - 35$

$-5x = -35$

$x = 7$

Sub $x = 7$ into ②

$y = -3x + 9$

$y = -3(7) + 9$

$= -21 + 9$

$= -12$

∴ Centre is $(7, -12)$

19. $P(10, 9)$
$Q(-2, 1)$
$R(-6, -15)$

① $m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{9 - 1}{10 - (-2)}$
$= \frac{8}{12}$
$= \frac{2}{3}$
$m_\perp = -3/2$

② $m_{QR} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{1 - (-15)}{-2 - (-6)}$
$= \frac{16}{4}$
$= 4$
$m_\perp = -1/4$

③ $M_{PQ} = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
$= M\left(\frac{-2 + 10}{2}, \frac{1 + 9}{2}\right)$
$= M(4, 5)$

④ $M_{QR} = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
$= M\left(\frac{-6 + (-2)}{2}, \frac{-15 + 1}{2}\right)$
$= M(-4, -7)$

⑤ $y = mx + b$
$5 = (-\frac{3}{2})(4) + b$
$5 = -12/2 + b$
$5 = -6 + b$
$b = 11$
$y = -\frac{3}{2}x + 11$

⑥ $y = mx + b$
$-7 = (\frac{-1}{4})(-4) + b$
$-7 = +4/4 + b$
$-7 = 1 + b$
$b = -8$
$y = -1/4x - 8$

⑦ POI: $y = -3/2x + 11$ ①

$y = -1/4x - 8$ ②

①×2: $2y = -3x + 22$ ③

②×4: $4y = -x - 32$ ④

③×−2 $-4y = 6x - 44$ ⑤

④+⑤ $0 = 5x - 76$

$\frac{-5x}{-5} = \frac{-76}{-5}$

$x = 15.2$

Sub $x = 15.2$ into ①

$y = -\frac{3}{2}(15.2) + 11$

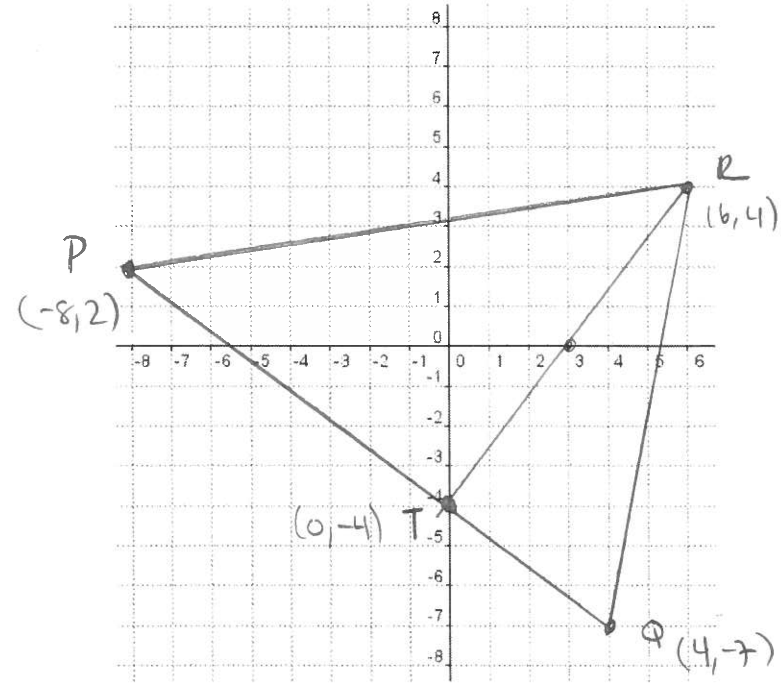
$= -1.5(15.2) + 11$

$= -22.8 + 11$

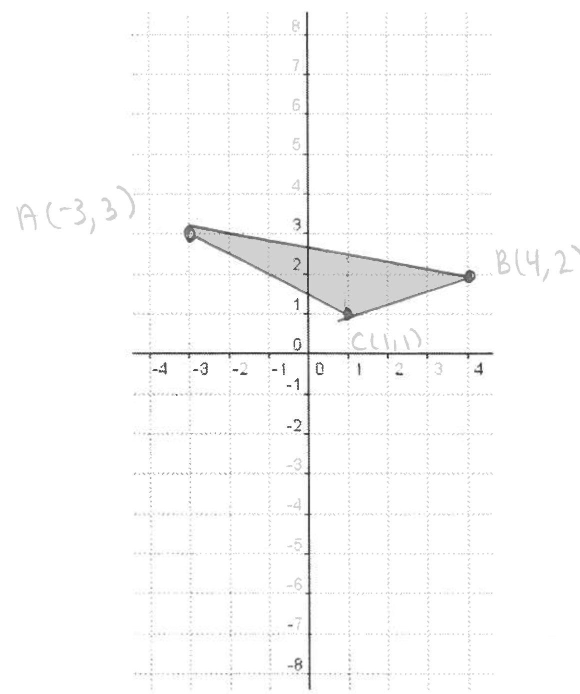
$= -11.8$

∴ Centre is $(15.2, -11.8)$

13. Determine the area of the triangle with vertices at P (–8, 2), Q (4, –7), and R (6, 4).



14. Determine the area of the triangle with vertices at A (–3, 3), B (4, 2), and C (1, 1).

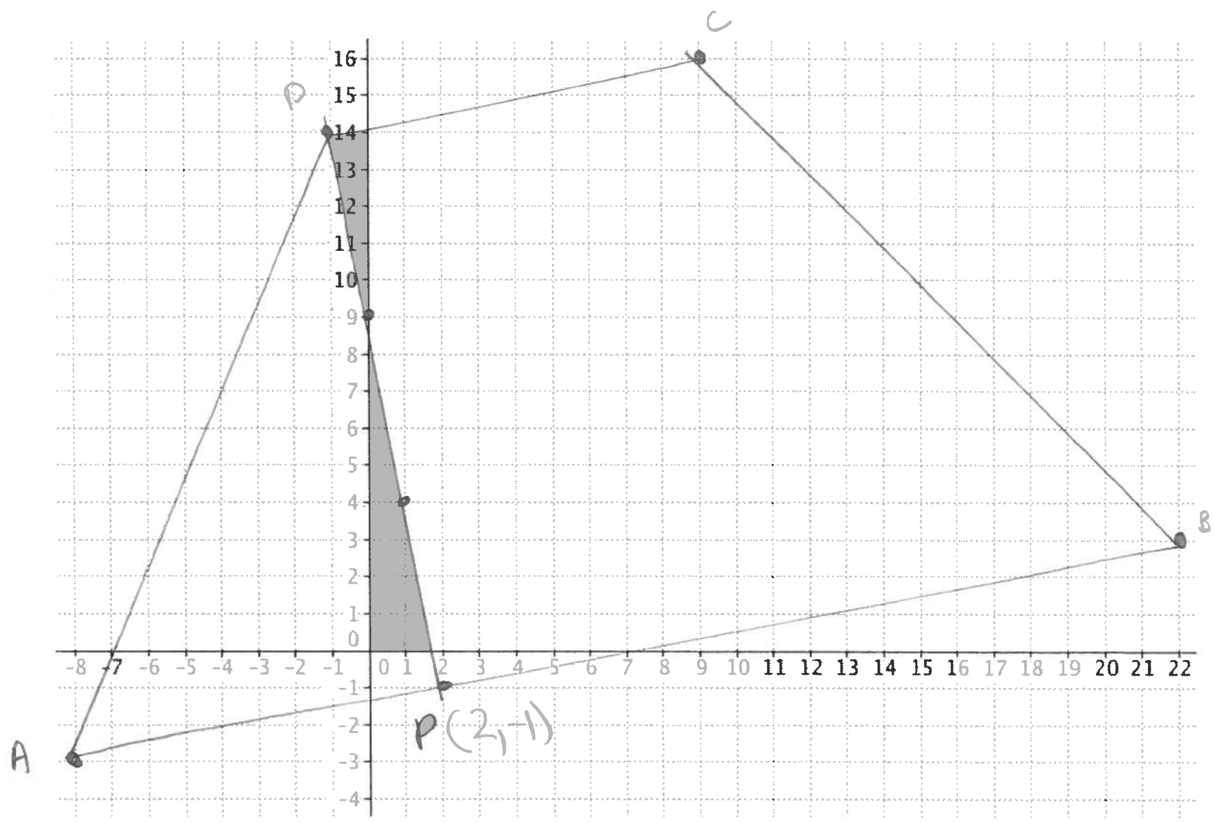


9. Triangle ABC has vertices A (3, 5), B (2, 3), and C (5, 2). Find the equation of the altitude from A to BC.

10. Find the shortest distance between the point (6, 5) and the line $7x + y + 23 = 0$.

11. Classify the following triangles as scalene or isosceles or equilateral.
    a. Triangle PQR with vertices at P (-1, 4), Q (8, 1), and R (1, -2).
    b. Triangle KLM with vertices at K (2, 5), L (4, 1), and M (6, -3).

12. Determine the area of the trapezoid with vertices A (–8, –3), B (22, 3), C (9, 16), and D (–1, 14).

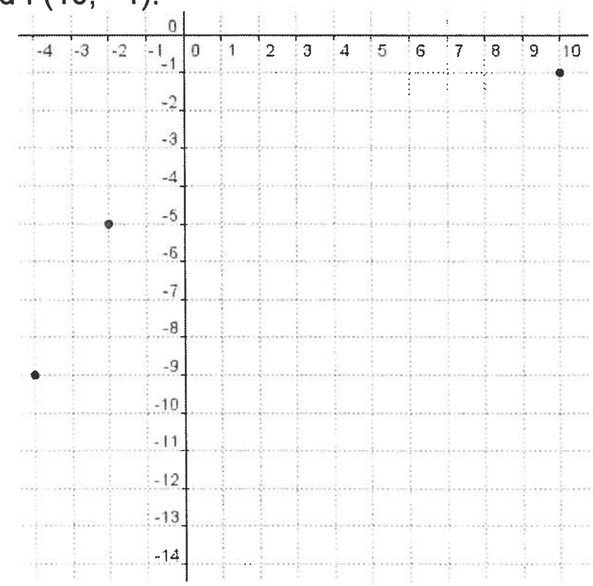


15. Given a quadrilateral with vertices at S (1, 2), T (3, 5), U (6, 7), and V (4, 4), verify that the quadrilateral is a parallelogram. Justify your answer.

16. Given a quadrilateral with vertices at A (1, 2), B (2, 5), C (5, 4), and D (4, 1), prove that the quadrilateral is a square. Justify your answer.

17. Given a quadrilateral with vertices at C (-1, 3), H (6, 4), A (4, -1), and D (-3, -2), determine if the quadrilateral is a parallelogram or trapezoid or square or rectangle.

18. Determine geometrically the centre of the circle that passes through the points G (–4, –9), H (–2, –5), and I (10, –1).



19. Determine algebraically the centre of the circle that passes through P (10, 9), Q (–2, 1), and R (–6, –15).