MPM2D – Exam Review – Unit 4

Unit 4 – The Quadratic Equation and Optimization

1. Complete the square to convert each of the following quadratic relations into vertex form.

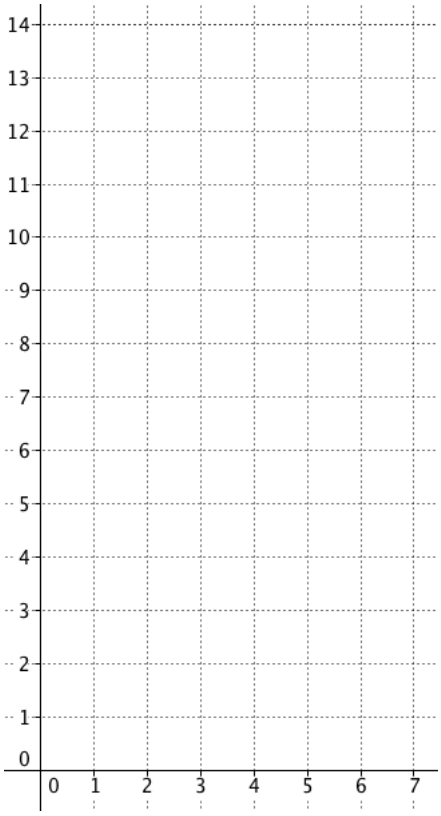
   a. $y = x^2 + 8x + 15$

   b. $y = 2x^2 + 12x - 3$

   c. $y = -5x^2 + 20x + 2$

   d. $y = -\frac{1}{2}x^2 + 12x - 7$

2. Complete the square to convert the quadratic relation, $y = -3x^2 + 30x - 63$, into vertex form. Graph the parabola and label the vertex and x-intercepts.



3. Determine the vertex and x-intercepts of the quadratic relation, $y = -4x^2 - 24x - 30$, by completing the square and solving by opposite operations.

4. Determine the vertex and x-intercepts of the quadratic relation, $y = 9x^2 - 36$, by opposite operations.

5. Determine the vertex and x-intercepts of the quadratic relation, $y = x^2 - 8x + 15$, by factoring.

6. Determine the vertex and x-intercepts of the quadratic relation, $y = 8x^2 - 26x - 15$, by factoring.

7. Determine the vertex and x-intercepts of the quadratic relation, $y = 3x^2 - 11x + 7$, by the quadratic formula.

8. Solve each of the following quadratic equations by factoring or quadratic formula or opposite operations.

a. $0 = 2(x+5)^2 - 1$	b. $0 = 2x^2 - 4$
c. $0 = 6x^2 - 11x - 10$	d. $x^2 + x = 72$
e. $5x^2 = 8x + 4$	f. $0 = -\frac{1}{2}(x+1)^2 + 5$
g. $0 = 4x^2 - 25$	h. $\frac{5x^2}{2} - \frac{3x}{2} = \frac{1}{4}$
i. $0 = x^2 - 13x + 22$	j. $\frac{3x^2}{2} - 4x = 8$
k. $0.75x^2 + 2.5x + 2 = 0$	l. $(2x-1)(x-5) = 0$

9. A glassworks that makes lead-crystal bowls has a daily production cost *C* in dollars given by the relation $C = 0.2b^2 - 10b + 650$ where *b* is the number of bowls made.
    a. How many bowls should be made to minimize the production cost?
    b. What is the cost when this many bowls are made?

10. The height, in metres, of a projectile is given by the equation $h = -4.9(t - 2.7)^2 + 41$, where t is the time in seconds.
    a. What is the maximum height of the projectile?
    b. What time does the projectile reach its maximum?
    c. What was the initial height of the projectile?
    d. When does the projectile hit the ground?

11. The area of the front cover of a daily journal is 273 cm^2 and the length is 8 cm greater than the width. What are the dimensions of the cover?

12. A farmer wants to make a rectangular corral along the side of a large barn and has enough materials for 60 m of fencing. Only three sides must be fenced since the barn wall will form the fourth side.
    a. What width will result in maximum area?
    b. What length will result in maximum area?
    c. What is the maximum area of the corral?

13. Two consecutive integers are added. The square of their sum is 361. What are the integers?

14. They hypotenuse of a right triangle measures 20 cm. The sum of the lengths of the other two sides is 28 cm. Find the lengths of the two sides.

15. A rectangular field is to be enclosed by a fence and divided into four smaller rectangular fields by two fences parallel to one of the sides. If there is 1200 m of fence, find the dimensions of the field giving maximum area.

16. The sum of the squares of three consecutive integers is 149. Find the values of the integers.

17. A rectangular lawn is to be made around a house. The dimensions of the house are 15 m by 25 m and the lawn will be a uniform width and its area will be 4 times that of the house. Find the width of the lawn, correct to two decimal places.

18. A playground that measures 60 m by 40 m is to be doubled in area by extending each side an equal amount. By how much should each side be extended?

19. A picture 20 cm by 20 cm is centrally mounted on a rectangular frame with total area three times the area of the picture. Assuming equal margins for all four sides, find the width of the margin.

20. A new car has a fuel consumption rate modeled by the equation $c = 0.0016v^2 - 0.2v + 11$ where *c* represents the rate at which fuel is consumed (in litres per hundred kilometres) and *v* represents the driving speed (in kilometres per hour).
    a. What driving speed will minimize fuel consumption?
    b. What is the minimum fuel consumption rate?
    c. What is the fuel consumption rate when driving at 115 km/h?

21. The height, h (in metres), of a snowboarder above the ground depends on how far the snowboarder has travelled horizontally, d (in metres), from the edge of a jump. The height can be modeled by $h = -0.1d^2 + 0.6d + 2.4$.
    a. Determine how far the snowboarder travelled horizontally before landing.
    b. Determine the maximum height of the snowboarder above the ground.

Answer Key

Unit 4

1a) $(x+4)^2 - 1$ b) $2(x+3)^2 - 21$ c) $-5(x-2)^2+22$ d) $y = -\frac{1}{2}(x-6)^2 + 65$

2) $-3(x-5)^2+12$ V (5, -12) Multiply the step pattern by -3

3) V (-3, 6) $x_1 \approx 1.78$ $x_2 \approx -4.22$

4) V (0, -36) $x_1 = 2$ $x_2 = -2$

5) V (4, -1) $x_1 = 3$ $x_2 = 5$

6) $V\left(\frac{13}{8}, \frac{289}{8}\right)$ $x_1 = \frac{15}{4}$ $x_2 = -\frac{1}{2}$

7) $V\left(\frac{11}{6}, -\frac{161}{18}\right)$ $x_1 \approx 2.85$ $x_2 \approx 0.82$

8a) $x_1 \approx -4.75$, $x_2 \approx -5.25$

b) $x_1 = \sqrt{2}$, $x_2 = -\sqrt{2}$

c) $x_1 = \frac{5}{2}$, $x_2 = -\frac{2}{3}$

d) $x_1 = 8$, $x_2 = -9$

e) $x_1 = -\frac{2}{5}$, $x_2 = 2$

f) $x_1 \approx 2.16$, $x_2 \approx -4.16$

g) $x = \frac{5}{2}$

h) $x_1 \approx 1.47$, $x_2 \approx -0.27$

i) $x_1 = 11$, $x_2 = 2$

j) $x_1 = -\frac{4}{3}$, $x_2 = 4$

k) $x_1 = -\frac{4}{3}$, $x_2 = -2$

l) $x_1 = \frac{1}{2}$, $x_2 = 5$

9a) 25 bowls b) C = $525

10a) 41 m b) 2.7 sec c) 5.279 m d) 5.59 sec

11) 21cm by 13cm 12a) 15 m b) 30 m c) 450 m^2

13) -9, -10 or 9, 10 14) 12cm by 16cm 15) 300 m by 150 m

16) -8, -7, -6 or 6, 7, 8 17) 11.79 m 18) 10 m

19) 7.32 cm 20a) 62.5 km/h b) 4.75 L/100 km c) 9.16 L/100 km

21a) 8.74 m b) 3.3 m