MPM2D – Exam Review – Unit 4

Unit 4 – The Quadratic Equation and Optimization

1. Complete the square to convert each of the following quadratic relations into vertex form.

   a. $y = x^2 + 8x + 15$

   b. $y = 2x^2 + 12x - 3$

   c. $y = -5x^2 + 20x + 2$

   d. $y = -\frac{1}{2}x^2 + 12x - 7$

2. Complete the square to convert the quadratic relation, $y = -3x^2 + 30x - 63$, into vertex form. Graph the parabola and label the vertex and x-intercepts.

$y = -3x^2 + 30x - 63$

$= -3(x^2 - 10x) - 63$

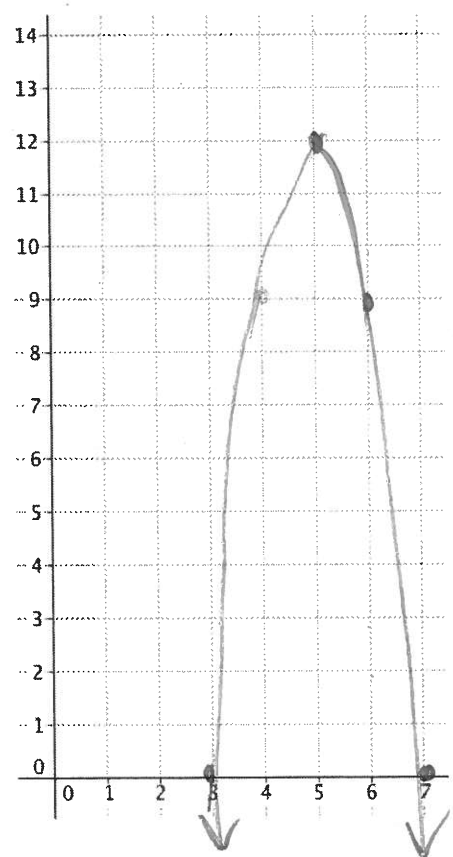
$= -3(x^2 - 5x - 5x + 25 - 25) - 63$

$= -3(x^2 - 5x - 5x + 25) + 75 - 63$

$= -3(x - 5)^2 + 12$

V (5, 12)

x	y
1	1(-3) = -3
2	4(-3) = -12
3	9(-3) = -27



3. Determine the vertex and x-intercepts of the quadratic relation, $y = -4x^2 - 24x - 30$, by completing the square and solving by opposite operations.

4. Determine the vertex and x-intercepts of the quadratic relation, $y = 9x^2 - 36$, by opposite operations.

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1. $y = x^2 + 8x + 15$
$= \underline{x^2 + 4x + 4x + 16} - 16 + 15$
$= (x+4)^2 - 1$

b. $y = 2x^2 + 12x - 3$
$= 2(x^2 + 6x) - 3$
$= 2(x^2 + 3x + 3x + 9 - 9) - 3$
$= 2(x^2 + 3x + 3x + 9) - 18 - 3$
$= 2(x+3)^2 - 21$

c. $y = -5x^2 + 20x + 2$
$= -5(x^2 - 4x) + 2$
$= -5(x^2 - 2x - 2x + 4 - 4) + 2$
$= -5(x^2 - 2x - 2x + 4) + 20 + 2$
$= -5(x-2)^2 + 22$

d. $y = -\frac{1}{2}x^2 + 12x - 7$
$= -\frac{1}{2}(x^2 - 24x) - 7$
$= -\frac{1}{2}(x^2 + 12x - 12x + 144 - 144) - 7$
$= -\frac{1}{2}(x^2 - 12x - 12x + 144) + 72 - 7$
$= -\frac{1}{2}(x-12)^2 + 65.$

2. See handout

3. $y = -4x^2 - 24x - 30$
$= -4(x^2 + 6x) - 30$
$= -4(x^2 + 3x + 3x + 9 - 9) - 30$
$= -4(x^2 + 3x + 3x + 9) + 36 - 30$
$= -4(x+3)^2 + 6$

$\boxed{\therefore V(-3, 6) \text{ and } x_1 \doteq 1.78 \text{ and } x_2 \doteq -4.22}$

x-int : y = 0
$0 = -4(x+3)^2 + 6$
$\frac{-6}{-4} = \frac{-4(x+3)^2}{-4}$
$1.5 = (x+3)^2$
$\sqrt{1.5} = \sqrt{(x+3)^2}$
$\pm 1.22 \doteq x + 3$
$x + 3 \doteq 1.22$ $\quad$ $x + 3 \doteq -1.22$
$x \doteq -1.78$ $\quad$ $x \doteq -4.22$

4. $y = 9x^2 - 36.$
x-int : y = 0
$0 = 9x^2 - 36$
$\frac{36}{9} = \frac{9x^2}{9}$
$4 = x^2$
$\sqrt{4} = \sqrt{x^2}$
$x = \pm 2$
$\boxed{V(0, -36), x_1 = 2, x_2 = -2}$

5. $y = x^2 - 8x + 15$
$y = (x-3)(x-5)$
$x_1 = 3 \quad x_2 = 5$

$x = \frac{3+5}{2}$ $\quad y =$
$= 8/2$
$= 4$

$y = x^2 - 8x + 15$
$= (4)^2 - 8(4) + 15$
$= 16 - 32 + 15$
$= -1$

$\boxed{V(4, -1) \quad x_1 = 3 \quad x_2 = 5}$

6. $y = 8x^2 - 26x - 15$

$= (4x - 15)(2x + 1)$

$x_1 = \frac{15}{4} \quad x_2 = -\frac{1}{2}$

$P = -120 \quad S = -26$

1, −120
2, −60
3, −40
$\boxed{4, -30}$

$x = \frac{15/4 + (-1/2)}{2}$

$= \frac{15/4 - 2/4}{} \div 2/1$

$= 13/4 \cdot 1/2$

$= 13/8$

$y = 8x^2 - 26x - 15$

$= 8(13/8)^2 - 26(13/8) - 15$

$= \frac{8}{1}\left(\frac{169}{64}\right) - \frac{338}{8} - \frac{15}{1}$

$= \frac{1352}{64} - \frac{2704}{64} - \frac{960}{64}$

$= \frac{-2312}{64}$

$= 289/8$

$$\boxed{\begin{array}{l} \therefore V(13/8, 289/8) \\ x_1 = 15/4 \\ x_2 = -1/2 \end{array}}$$

7. $y = 3x^2 - 11x + 7$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-11) \pm \sqrt{(-11)^2 - 4(3)(7)}}{2(3)}$

$= \frac{11 \pm \sqrt{121 - 84}}{6}$

$x_1 = \frac{11 + \sqrt{37}}{6} \quad x_2 = \frac{11 - \sqrt{37}}{6}$

$x_1 \doteq 2.85 \quad x_2 \doteq 0.82$

Vertex

$x = \frac{-b}{2a}$

$= \frac{-(-11)}{2(3)}$

$= \frac{11}{6}$

$y = 3x^2 - 11x + 7$

$= 3\left(\frac{11}{6}\right)^2 - 11\left(\frac{11}{6}\right) + 7$

$= 3\left(\frac{121}{36}\right) - \frac{121}{6} + \frac{7}{1}$

$= \frac{363}{36} - \frac{726}{36} + \frac{42}{36}$

$= \frac{-322}{36}$

$= -\frac{161}{18}$

$$\boxed{\begin{array}{l} \therefore V\left(\frac{11}{6}, \frac{-161}{18}\right) \\ x_1 \doteq 2.85 \\ x_2 \doteq 0.82 \end{array}}$$

8. $0 = +2(x+5)^2 - 1$

$\frac{1}{2} = \frac{2(x+5)^2}{2}$

$0.5 = (x+5)^2$

$\sqrt{0.5} = \sqrt{(x+5)^2}$

$\pm 0.25 = x+5$

$x+5 = 0.25$ $\quad x+5 = -0.25$

$x = -4.75$ $\quad x = -5.25$

b. $0 = 2x^2 \pm 4$ $\quad 2(x^2+2)$

$\frac{+4}{2} = \frac{2x^2}{2}$

$2 = x^2$

$x = \pm\sqrt{2}$

$x_1 = \sqrt{2}$ $\quad x_2 = -\sqrt{2}$

c. $0 = 6x^2 - 11x - 10$ $\quad P = -60$ $S = -11$

$0 = (2x-5)(3x+2)$ $\quad -15, 4$

$x_1 = \frac{5}{2}$ $\quad x_2 = -\frac{2}{3}$

d. $x^2 + x = 72$ $\quad P = -72$ $S = 1$

$x^2 + x - 72 = 0$ $\quad 9, -8$

$(x-8)(x+9) = 0$

$x_1 = 8$ $\quad x_2 = -9$

e. $5x^2 = 8x + 4$ $\quad P = -20$ $S = -8$

$5x^2 - 8x - 4 = 0$ $\quad -10, 2$

$(5x+2)(x-2) = 0$

$x_1 = -\frac{2}{5}$ $\quad x_2 = 2$

f. $0 = -\frac{1}{2}(x+1)^2 + 5$

$\frac{-5}{-1/2} = \frac{-1/2(x+1)^2}{-1/2}$

$10 = (x+1)^2$

$\sqrt{10} = \sqrt{(x+1)^2}$

$\pm 3.16 \doteq (x+1)$

$x+1 \doteq 3.16$ $\quad x+1 \doteq -3.16$

$x_1 \doteq 2.16$ $\quad x_2 \doteq -4.16$

fav* :) g. $0 = 4x^2 - 25$

$0 = (2x-5)(2x+5)$

$x_1 = \frac{5}{2}$

h. $\frac{5x^2}{2} - \frac{3x}{2} = \frac{1}{4}$ $\quad \times 4$

$\frac{20x^2}{2} - \frac{12x}{2} = \frac{4}{4}$

$10x^2 - 6x = 1$

$10x^2 - 6x - 1 = 0$ $\quad P = -10$ $S = -6$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(10)(-1)}}{2(10)}$

$= \frac{6 \pm \sqrt{36 + 40}}{20}$

$\rightarrow x_1 = \frac{6+\sqrt{76}}{10}$ $\quad x_2 = \frac{6-\sqrt{76}}{10}$

$x_1 \doteq 1.47$ $\quad x_2 = -0.27$

i $0 = x^2 - 13x + 22$ $P=22$ $S=-13$

$0 = (x-11)(x-2)$ $-11, -2$

$x_1 = 11$ $x_2 = 2$

j) $\frac{3x^2}{2} - 4x = 8$

$3x^2 - 8x = 16$ $P=-48$ $S=-8$

$3x^2 - 8x - 16 = 0$ $-12, 4$

$(3x+4)(x-4) = 0$

$x_1 = -\frac{4}{3}$ $x_2 = 4$

k. $0.75x^2 + 2.5x + 2 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-2.5 \pm \sqrt{(2.5)^2 - 4(0.75)(2)}}{2(0.75)}$

$= \frac{-2.5 \pm \sqrt{6.25 - 6}}{1.5}$

$= \frac{-2.5 \pm \sqrt{0.25}}{1.5}$

$x_1 = \frac{-2.5 + \sqrt{0.25}}{1.5}$ $x_2 = \frac{-2.5 - \sqrt{0.25}}{1.5}$

$x_1 = -1.33$ $x_2 = -2$

$(-4/3)$

l) $(2x-1)(x-5) = 0$

$x_1 = \frac{1}{2}$ $x_2 = 5$

9. $C = 0.2b^2 - 10b + 650$

a) minimum → find vertex

$x = \frac{-b}{2a}$

$= \frac{-(-10)}{2(0.2)}$

$= \frac{10}{0.4}$

$= 25$

$\therefore$ 25 bowls

b) $C = 0.2b^2 - 10b + 650$

$= 0.2(25)^2 - 10(25) + 650$

$= 0.2(625) - 250 + 650$

$= 125 - 250 + 650$

$= 525$

$\therefore$ \$525.

10. $h = -4.9(t-2.7)^2 + 41$

a) max height is 41 m.

b) projectile reaches max height at 2.7 sec

c) initial height $t = 0$

$$h = -4.9(t-2.7)^2 + 41$$
$$= -4.9(0-2.7)^2 + 41$$
$$= -4.9(-2.7)^2 + 41$$
$$= -4.9(7.29) + 41$$
$$= -35.721 + 41$$
$$= 5.279 m$$

It's initial height is 5.279m.

d. hit ground $h = 0$

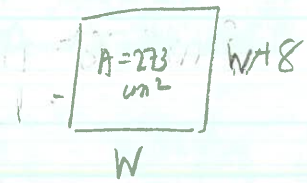
$$h = -4.9(t-2.7)^2 + 41$$
$$0 = -4.9(t-2.7)^2 + 41$$
$$\frac{-41}{-4.9} = \frac{-4.9(t-2.7)^2}{-4.9}$$
$$8.37 \doteq (t-2.7)^2$$
$$\sqrt{8.37} \doteq \sqrt{(t-2.7)^2}$$
$$\pm 2.89 \doteq t - 2.7$$

$t - 2.7 = 2.89$ $\quad$ $t - 2.7 = -2.89$

$t = 5.59$ $\quad$ $t = -0.19$

imp

∴ It hits the ground at 5.59 sec.

11. 

Let w and w+8 be the dimensions.

$A = lw$

$273 = (w+8)(w)$

$273 = w^2 + 8w$

$0 = w^2 + 8w - 273$

$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$w = \frac{-8 \pm \sqrt{8^2 - 4(1)(-273)}}{2(1)}$

$= \frac{-8 \pm \sqrt{64 + 1092}}{2}$

$w_1 = \frac{-8 + \sqrt{1156}}{2}$ $\quad w_2 = \frac{-8 - \sqrt{1156}}{2}$

$w_1 = \frac{-8 + 34}{2}$ $\quad w_2 = \frac{-8 - 34}{2}$

$= 26/2$ $\quad = -42/2$

$= 13$ $\quad = -21$ IMP

$w = 13$

$l = w + 8$

$= 21$

∴ The journal is 21cm by 13cm.

max area / find vertex

12. 

Let l & w represent the dimensions.

$P = 2w + l$

$60 = 2w + l$

$l = 60 - 2w$

$A = lw$

$= (60 - 2w)(w)$

$= 60w - 2w^2$

$= -2w^2 + 60w$

$w = \frac{-b}{2a}$

$= \frac{-60}{2(-2)}$

$= \frac{-60}{-4}$

$\boxed{w = 15m}$

$l = 60 - 2w$

$= 60 - 2(15)$

$= 60 - 30$

$= 30m$

$A = -2w^2 + 60w$

$= -2(15)^2 + 60(15)$

$= -2(225) + 900$

$= -450 + 900$

$= 450m^2$

∴ The dimensions should be 15m by 30m to create a maximum area of 450m².

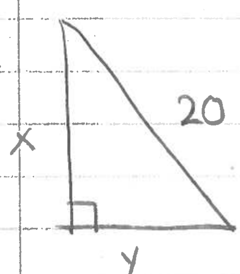
13. Let x and $x+1$ be the #s.

$$[x+(x+1)]^2 = 361$$
$$(2x+1)^2 = 361$$
$$(2x+1)(2x+1) = 361$$
$$4x^2+2x+2x+1-361 = 0$$
$$\frac{4x^2}{4}+\frac{4x}{4}-\frac{360}{4} = \frac{0}{4}$$
$$x^2+x-90 = 0$$ $P=90$ $S=1$
$$(x-9)(x+10) = 0$$ $-9, 10$
$$x=9 \quad x=-10$$

$x=9$ $x=-10$
$x+1=10$ $x+1=-9$

∴ The #s are $9 \& 10$ or $-9 \& -10$.

14.



$x+y=28$
$x=28-y$

Let y and $28-y$ be the side lengths.

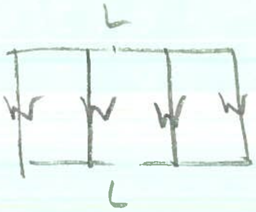
$(28-y)(28-y)$
$=784-28y-28y+y^2$
$=y^2-56y+784$

$$a^2+b^2 = c^2$$
$$x^2+y^2 = 20^2$$
$$(28-y)^2+y^2 = 400$$
$$y^2-56y+784+y^2-400 = 0$$
$$\frac{2y^2}{2}-\frac{56y}{2}+\frac{384}{2} = \frac{0}{2}$$
$$y^2-28y+192 = 0$$
$$y = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$= \frac{-(-28) \pm \sqrt{(-28)^2-4(1)(192)}}{2(1)}$$
$$= \frac{28 \pm \sqrt{784-768}}{2}$$
$$y_1 = \frac{28+\sqrt{16}}{2} \quad y_2 = \frac{28-\sqrt{16}}{2}$$
$$y_1 = \frac{28+4}{2} \quad y_2 = \frac{28-4}{2}$$
$$= 16 \quad = 12$$

$y=16$
$x=28-y$
$=28-16$
$=12$

$y=12$
$x=28-12$
$=16$

∴ The side lengths are 12 cm and 16 cm.

15. 

Let L & W represent the dimensions.

$P = 2L + 4W$

$\frac{1200}{2} = \frac{2L}{2} + \frac{4W}{2}$

$600 = L + 2W$

$L = 600 - 2W$

$A = LW$

$= (600 - 2W)(W)$

$= 600W - 2W^2$

$= -2W^2 + 600W$

$W = \frac{-b}{2a}$

$= \frac{-600}{2(-2)}$

$= 150$

$L = 600 - 2W$

$= 600 - 2(150)$

$= 300$

∴ The dimensions for max area are 300m by 150m

16. Let x, x+1, and x+2 be the #s.

$(x+1)(x+1) = x^2 + x + x + 1$

$(x+2)(x+2) = x^2 + 2x + 2x + 4$

$(x)^2 + (x+1)^2 + (x+2)^2 = 149$

$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 149$

$3x^2 + 6x + 5 - 149 = 0$

$\frac{3x^2}{3} + \frac{6x}{3} - \frac{144}{3} = \frac{0}{3}$

$x^2 + 2x - 48 = 0$ $\quad P = -48 \quad S = 2$

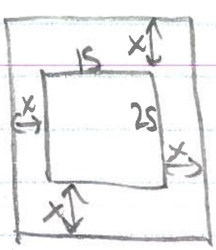
$(x+8)(x-6) = 0$

$x_1 = -8 \qquad x_2 = 6$

$x+1 = -7 \qquad x+1 = 7$

$x+2 = -6 \qquad x+2 = 8$

∴ The #s are 6, 7, 8 or −6, −7, −8.

17. 

Let x be the width of the house

$A_h = lw$

$= 15(25)$

$= 375m^2$

$A_L = 4A_h$

$= 4(375)$

$= 1500m^2$

$A_T = A_h + A_L$

$= 375 + 1500$

$= 1875m^2$

$A_T = LW$

$1875 = (15 + 2x)(25 + 2x)$

$1875 = 375 + 30x + 50x + 4x^2$

$\frac{0}{4} = \frac{4x^2}{4} + \frac{80x}{4} - \frac{1500}{4}$

$0 = x^2 + 20x - 375$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-20 \pm \sqrt{(20)^2 - 4(1)(-375)}}{2(1)}$

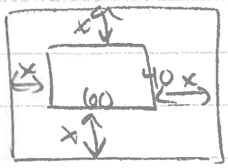
$= \frac{-20 \pm \sqrt{400 + 1500}}{2}$

$x_1 = \frac{-20 + \sqrt{1900}}{2} \qquad x_2 = \frac{-20 - \sqrt{1900}}{2}$

$x_1 = 11.79 \qquad x_2 = -31.79$ imp

∴ The lawn will be 11.79m wide

18.



$A_i = lw$
$= 40(60)$
$= 2400 m^2$

$A_T = 2A_i$
$= 2(2400)$
$= 4800 m^2$

Let x be the uniform width added.

$A_T = LW$
$4800 = (60+2x)(40+2x)$
$4800 = 2400 + 120x + 80x + 4x^2$
$\frac{0}{4} = \frac{4x^2}{4} + \frac{200x}{4} - \frac{2400}{4}$
$0 = x^2 + 50x - 600$

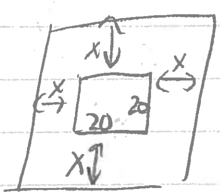
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$= \frac{-50 \pm \sqrt{(50)^2 - 4(1)(-600)}}{2(1)}$
$= \frac{-50 \pm \sqrt{2500+2400}}{2}$
$= \frac{-50 \pm \sqrt{4900}}{2}$

$x_1 = \frac{-50+70}{2}$ $x_2 = \frac{-50-70}{2}$

$x_1 = 10$ $x_2 = -60$ IMP

∴ 10m will be added to each side to double the area.

19.



$A_i = lw$
$= 20(20)$
$= 400 m^2$

$A_T = 3A_i$
$= 3(400)$
$= 1200 m^2$

Let x be the uniform width of the margin.

$A_T = lw$
$1200 = (20+2x)(20+2x)$
$1200 = 400 + 40x + 40x + 4x^2$
$\frac{0}{4} = \frac{4x^2}{4} + \frac{80x}{4} - \frac{800}{4}$
$0 = x^2 + 20x - 200$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$= \frac{-20 \pm \sqrt{(20)^2 - 4(1)(-200)}}{2(1)}$
$= \frac{-20 \pm \sqrt{400+800}}{2}$

$x_1 = \frac{-20+\sqrt{1200}}{2}$ $x_2 = \frac{-20-\sqrt{1200}}{2}$

$x_1 \doteq 7.32$ $x_2 \doteq -27.32$

∴ The margin will be 7.32cm wide.

20. $C = 0.0016v^2 - 0.2v + 11$

a. minimize = vertex

$v = \frac{-b}{2a}$

$= \frac{-(-0.2)}{2(0.0016)}$

$= 62.5$ km/h

b) $C = 0.0016v^2 - 0.2v + 11$

$= 0.0016(62.5)^2 - 0.2(62.5) + 11$

$= 0.0016(3906.25) - 12.5 + 11$

$= 6.25 - 12.5 + 11$

$= 4.75$ L/100 km

c) $C = 0.0016v^2 - 0.2v + 11$

$= 0.0016(115)^2 - 0.2(115) + 11$

$= 0.0016(13225) - 23 + 11$

$= 9.16$ L/100 km

21. $h = -0.1d^2 + 0.6d + 2.4$

a) landing: $h = 0$
$\downarrow$
x-int

$h = -0.1d^2 + 0.6d + 2.4$

$0 = -0.1d^2 + 0.6d + 2.4$

$d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-0.6 \pm \sqrt{(0.6)^2 - 4(-0.1)(2.4)}}{2(0.1)}$

$= \frac{-0.6 \pm \sqrt{0.36 + 0.96}}{-0.2}$

$d_1 = \frac{-0.6 + \sqrt{1.32}}{-0.2}$ $d_2 = \frac{-0.6 - \sqrt{1.32}}{-0.2}$

$d_1 \doteq -2.74$ $d_2 \doteq 8.74$
$\downarrow$ imp

∴ He lands on the ground after travelling 8.74 m horizontally.

b. max height
$\downarrow$
vertex

$d = -b/2a$

$= \frac{-0.6}{2(-0.1)}$

$= 3$

$h = -0.1d^2 + 0.6d + 2.4$

$= -0.1(3)^2 + 0.6(3) + 2.4$

$= -0.1(9) + 1.8 + 2.4$

$= 3.3$

∴ The max height is 3.3m.