Determining the x–intercepts of a Quadratic Relation

We began our study of quadratic relations by considering parabolas in vertex form:

$y = a(x-h)^2 + k$ has a vertex where x =

has x-intercepts where y =

Example – Determine the x-intercepts of the quadratic relation $y = -6(x-2.5)^2 + 47$.
Sketch its graph and label the coordinates of important points.

Factoring lets us convert quadratic relations from standard form to factored form:

$y = a(x - r)(x - s)$ has x-intercepts where x =

has a vertex where x =

This provides a “shortcut” for determining x-intercepts when factoring is possible.

Example – Determine the x-intercepts of the quadratic relation $y = -6x^2 - 24x + 30$.
Sketch its graph and label the coordinates of important points.

Completing the square lets us convert from standard form to vertex form.

This provides a method for determining x-intercepts when factoring is not possible.

Example – Determine the x-intercepts of the quadratic relation $y = -6x^2 + 60x - 87$.
Sketch its graph and label the coordinates of important points.

We can now solve problems involving projectile motion modeled in standard form.

Example – A golf ball is launched with an initial speed of 48 km/h at an angle of 81° above horizontal from an initial height of 8 m above the fairway.
Its height, h (in metres), above the fairway can be modeled with respect to its horizontal distance, d (in metres), using the following quadratic relation:

$$h = -d^2 + 6d + 8$$

a) Determine the maximum height of the golf ball above the fairway.
b) Determine when the golf ball will hit the fairway.
c) Determine when the golf ball will be 11 m above the fairway.
d) Illustrate your answers on a sketch of the trajectory of the golf ball.

Homework:

1. Determine the x-intercepts of the quadratic relation $y = -3x^2 - 42x - 130$. Sketch its graph and label the coordinates of the vertex and x-intercepts.

2. Determine the x-intercepts of the quadratic relation $y = -5x^2 + 45x - 70$.

3. Determine the x-intercepts of the quadratic relation $y = -6(x - 1.8)^2 + 27$.

4. Determine the vertex and x-intercepts of $y = -4x^2 + 24x - 22$. Sketch the graph of this parabola to illustrate your answers.

5. A golf ball is launched with an initial speed of 64 km/h at an angle of 83° above horizontal from an initial height of 7 m above the fairway. Its height, h (in metres), above the fairway can be modeled with respect to its horizontal distance, d (in metres), using the following:

$$h = -d^2 + 8d + 7$$

   a) Determine the maximum height of the golf ball above the fairway.
   b) Determine when the golf ball will hit the fairway.
   c) Determine when the golf ball will be 15 m above the fairway.
   d) Illustrate your answers on a sketch of the trajectory of the golf ball.

Answers:

1. The x-intercepts are about (–4.62, 0) and (–9.38, 0). The vertex is (–7, 17).

2. The x-intercepts are (2, 0) and (7, 0).

3. The x-intercepts are about (3.92, 0) and (–0.32, 0).

4. The vertex is (3, 14). The x-intercepts are about (1.13, 0) and (4.87, 0).

5. a) The maximum height of the golf ball above the fairway was 23 m.

   b) The golf ball will hit the fairway after travelling 8.80 m horizontally.

   c) The golf ball will be 15 m above the fairway after travelling 1.17 m horizontally and again after travelling 6.83 m.

   d) Label the vertex (1.8, 27) and x-intercepts (1.17, 0) and (6.83, 0).