Completing the Square – Part 1

We can now convert a quadratic relation from standard form to factored form:

Factoring

$$y = ax^2 + bx + c \longrightarrow y = a(x - r)(x - s)$$

We can also locate the vertex using the axis of symmetry: $x = \dfrac{r + s}{2}$

Therefore, we can write the quadratic relation in vertex form: $y = a(x - h)^2 + k$

This process works well when a quadratic relation can be factored using integers.

Example – Factor the quadratic relation $y = 2x^2 + 4x - 30$ and determine its equation in vertex form. Sketch its graph and label all important points.

Next, we will develop a process for determining the vertex (and x-intercepts) when a quadratic relation cannot be factored using integers.

The first step in this process is learning how to complete the square in order to convert a quadratic relation from standard form to factored form:

$$y = ax^2 + bx + c \xrightarrow{\textit{Completing the Square}} y = a(x-h)^2 + k$$

We can begin by identifying the necessary coefficient c in order to make $x^2 + bx$ into a perfect square trinomial.

Example – Fill in the blanks below to create each perfect square trinomial.

a. $x^2 + 6x + c$

$= x^2 + 3x + 3x + \_\_\_\_\_$

$= (x + \_\_\_\_\_)(x + \_\_\_\_\_)$

$= (x + \_\_\_\_\_)^2$

c. $x^2 + 10x + c$

b. $x^2 - 8x + c$

d. $x^2 - 2x + c$

Completing the square means converting from standard form to factored form.

To Complete the Square for $ax^2 + bx + c$ **when** $a = 1$**:**

1. Separate the middle term into two equal parts: $bx = b_1x + b_1x$
2. Add and subtract the square of the equal part: $+b_1^2 - b_1^2$
3. Factor the perfect square portion (the first four terms).

Example - Complete the square for each of the quadratic relations below. Then state the vertex of each quadratic relation.

a. $y = x^2 - 8x + 7$

b. $y = x^2 - 2x + 5$

c. $y = x^2 + 6x + 7$

d. $y = x^2 + 10x - 1$

e. $y = x^2 + 12x - 4$

Homework – Please complete questions #2, 3, 4, 5, and 6 (no graphs) on p 270.